Testing for Aggregate Instability in an Open Economy

Yusuf Ömür Yılmaz*

Abstract

This paper empirically tests the Turkish economy's stability between 2010 and 2016 using a small open economy New-Keynesian model featuring a positive long-run inflation rate and sunspot shock under alternative monetary policy rules. We found that the probabilities of indeterminacy are very close to unity. We can thus conclude that the economy is unstable over this period primarily because the long-run inflation rate is in the higher single digits despite the Central Bank's weak response to inflation. This drove the economy into aggregate instability.

JEL Classification: E31, E32, E52, E58
Keywords: Aggregate Instability, Turkey's Economy, Long-run Inflation, New-Keynesian Model, Determinacy

Açık Bir Ekonomide Toplam İstikrarsızlığın Test Edilmesi

Özet

Bu makale, alternatif para politikası kuralları altında pozitif uzun vadeli enflasyon oranı ve güneş lekesi şoku içeren küçük bir açık ekonomi Yeni-Keynesyen modeli kullanarak 2010 ve 2016 yılları arasında Türk ekonomisinin istikrarını ampirik olarak test etmektedir. Belirsizlik olasılıklarının bire çok yakın olduğu bulunmuştur. Bu dönemde ekonominin istikrarsız olduğu sonucuna varılabilir çünkü Merkez Bankası'nın enflasyona zayıf tepki vermesine rağmen uzun vadeli enflasyon oranı yüksek tek hanelidir. Bu durum ekonomiyi genel istikrarsızlığa sürüklemiştir.

JEL Sınıflandırması: E31, E32, E52, E58
Anahtar Kelimeler: Toplam İstikrarsızlık, Türkiye Ekonomisi, Uzun Dönemli Enflasyon, Yeni Keynesyen Model, Belirlenebilirlik

*Department of Economics, University of Mardin Artuklu., 47200 Mardin, Turkey. E-mail: yusufomuryilmaz@artuklu.edu.tr, ORCID: 0000-0001-9486-1618

Boğaziçi Journal Review of Social, Economic and Administrative Studies, Vol. 36, no. 1 (2022), pp. 1-17, doi:10.21773/boun.36.1.1 Research Article | Received: 01.09.2021 - Accepted: 19.09.2022

1. Introduction

During the 1980s and 90s, the Turkish economy witnessed a long period of high, volatile inflation. Between the 2001 Financial Crisis and 2016, the economy experienced relatively stable macroeconomic circumstances (i.e., a relatively low inflation rate and output gap). Even so, the average CPI inflation rate remained at around 8%. Woodford (2003, p.254) notes, "[…] at least in the long-run, nominal interest rates should rise by more than the increase in the inflation rate". Higher long-run inflation rates should prompt stronger responses.[1] Otherwise, multiple possible values of macroeconomic variables (inflation rate, output, etc.) may emerge via two channels: sunspot shocks and the varying propagation mechanism of fundamental shocks. This situation is referred to as "the indeterminacy of monetary policy rules".

Özatay (2011) argues that in 2010 the Central Bank changed its monetary policy considerably. Gürkaynak et al. (2015) found that the Central Bank responded strongly to the pre-2010 inflation rate but responded weakly after 2010. Inflation expectations are thus not well-anchored in the Turkish economy. Considering Turkey's higher long-run inflation rate, this opens the possibility of indeterminacy in the Turkish economy. However, there is as yet no study analyzing the aggregate stability of the Turkish Economy in the context of rational expectations[2]. In this paper, we empirically discuss the indeterminacy of monetary policy rules in an otherwise small open economy New-Keynesian model over the period from 2010: Q2-2016: Q4.

Our study is based on the Small Open Economy model in Gali and Monacelli (2005).[3] We extend this model through three channels. First, we log-linearize equations, resetting prices and the Calvo Rule around positive long-run inflation rate (or trend inflation) while accounting for inflation indexation. Second, we incorporate two alternative monetary policy rules into the model. Third, we endogenize forecast error in the model, through the CPI inflation rate, by employing the methodology of Lubik and Schorfheide (2003,2004) together with Farmer et al.'s (2015) approach of allowing indeterminacy in the model.[4]

We discuss the aggregate stability of the economy through the determinacy probability approach. We find that, over the period, the probabilities of indeterminacy are near unity under alternative monetary policy rules. We can thus conclude that the economy is likely indeterminate over the period. The Turkish economy's main empirical features are higher long-run inflation, higher price rigidity, and a weak nominal interest rate response to inflation. To analyze some potential reasons behind the indeterminacy of these monetary policy rules, we discuss the effects of these empirical features over the determinacy region. For example, the determinacy region shrinks more at higher long-run inflation rates and with high price rigidity at positive long-run inflation rates. Additionally, trade openness aggravates the effect of long-run inflation rate on the determinacy region. Considering these features of the Turkish economy, the Central Bank responds weakly to increases in inflation rate and monetary policy rules are thus not strong enough to rule out indeterminacy. Self-fulfilling fluctuations in macroeconomic dynamics may occur.

[1] See Ascari and Sbordone (2014) for further details.
[2] There are some Dynamic Stochastic General Equilibrium (DSGE) studies on the Turkish Economy, including Çebi (2011), and Alp and Elekdag (2011).
[3] Zhang and Dai (2020); Araujo (2004); Llosa, L. and Tuesta (2008) ; De Fiore and Liu (2005) analyze determinacy in a small open economy.
[4] Farmer et al. (2015) show that models with different expectational errors provide similar results.

This paper is organized into sections. Section 2 describes the model economy. Section 3 presents data, calibration values, and priors. Section 4 presents an estimation strategy. Section 5 presents the results of empirical studies. Section 6 discusses possible reasons of indeterminacy. Section 7 presents a robustness analysis. Section 8 concludes the paper.

2. Model

The world economy comprises a continuum of small open economies indexed by $i \in [0,1]$. One such economy is termed the "home economy" while the rest are called "the rest of the world". Any policy taken in one of these small open economies does not influence the rest of the world. All small open economies have the same market structure, the same technology function and preferences. In the home economy, there is a representative household, a representative final good-producing firm, a continuum of intermediate goods-producing firms, and a central bank. Aside from the price setting behavior of intermediate firms, the model is identical to that of Gali and Monacelli (2005). Note that variables without any index correspond to home economy, ones with i index correspond to country i and ones with an asterisk superscript correspond to the world economy.

2.1. Household

A representative household aims to maximize the following period utility function

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{1}$$

subject to the period by period budget constraint shown in Equation 2.

$$P_t C_t + \mathrm{E}_t \tau_{t+1,t} B_{t+1} = B_t + W_t N_t + D_t \tag{2}$$

where σ is risk aversion, φ is inverse Frisch elasticity of labor supply, P_t is the Consumer Price Index (CPI), C_t is the composite consumption index, $\tau_{t+1,t}$ is the stochastic discount factor between time t and t+1, and it equals $\frac{1}{1+i_t}$, $B_{t+1}$is the nominal payoff in period t+1 of the portfolio held at the end of period t, W_t is the nominal wage, N_t is labour force. D_t is dividend and E_t is expectation operator at time t.

Optimality conditions are:

$$\tau_{t+1,t} = \beta E_t(\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}}) \tag{3}$$

$$w_t^r = \frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi} \tag{4}$$

where w_t^r is real wage and β is the intertemporal discount factor.

2.2. Firms

There is a continuum of intermediate goods-producing firms indexed by $j \in [0,1]$ in the home economy. A typical firm *j* in the home economy produces a differentiated good *j* with linear technology represented by the production function

$$Y_{j,t} = A_t N_{j,t} \tag{5}$$

where $Y_{j,t}$ is the output produced by firm *j*. A_t denotes technology and follows $A_{t-1}^{\rho_a} \exp(e_{a,t})$ where ρ_a is the persistence of shock and $e_{a,t}$ is the shock innovation. It is common to all firms. $N_{j,t}$ is labor force employed by firm *j*.

Aggregating Equation 5 over *j* leads to the following expression:

$$\int_0^1 Y_{j,t} dj = \int_0^1 A_t N_{j,t} dj = \int_0^1 \left(\frac{P_{H,j,t}}{P_{H,t}}\right)^{-\epsilon} dj \, Y_t = A_t N_t$$

$$Y_t = \frac{1}{z_t} A_t N_t \tag{6}$$

where $\int_0^1 \left(\frac{P_{H,j,t}}{P_{H,t}}\right)^{-\epsilon} dj = z_t$ is the price dispersion, $P_{H,t}$ is the domestic price index, $P_{H,j,t}$ is the price of good *j* and ϵ is the elasticity of substitution between differentiated goods.

Aggregate Price Dynamics

Each intermediate goods-producing firm follows Calvo's (1983) rule, according to which each firm updates its nominal priceswith a probability of $1-\theta$ and indexes its nominal price to the previous period's CPI inflation rate with a probability of θ. The rule is as follows:

$$P_{H,t} = \left[(1-\theta)\left(\bar{P}_{H,t}\right)^{1-\epsilon} + \theta\left(\pi_{t-1}^{\zeta} P_{H,t-1}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{7}$$

where $\bar{P}_{H,t}$ is the reset price and ζ is the degree of indexation.

$\pi_t = \frac{P_t}{P_{t-1}}$ is the CPI inflation rate between periods t and t-1, while $\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$ is the domestic inflation rate between periods t and t-1.

Optimal Price Setting

Intermediate firm *j* chooses reset price $\bar{P}_{H,t}$ to maximise the present value of its profit:

$$\max_{\bar{P}_{H,t}} E_t \sum_{k=0}^{\infty} \theta^k \left[\tau_{t+k,t}\left(\bar{P}_{H,t} Y_{j,t+k} - W_{t+k} N_{j,t+k}\right)\right] \tag{8}$$

subject to the demand constraint: $Y_{j,t+k} = \left(\frac{\Pi_{t+k-1,t-1}^{\zeta}}{P_{H,t+k}} \bar{P}_{H,t}\right)^{-\epsilon} Y_{t+k}$.

$$\bar{p}_{H,t} = \frac{\epsilon}{\epsilon-1}\frac{E_t\sum_{k=0}^{\infty}(\beta\theta)^k\Pi_{H,t+k,t}^{\epsilon+1}\Pi_{t+k,t}^{-1}\Pi_{t+k-1,t-1}^{-\zeta\epsilon}MC_{t+k}^{r}\,Y_{t+k}C_{t+k}^{-\sigma}}{E_t\sum_{k=0}^{\infty}(\beta\theta)^k\Pi_{H,t+k,t}^{\epsilon}\Pi_{t+k,t}^{-1}\Pi_{t+k-1,t-1}^{-\zeta\epsilon}\,Y_{t+k}C_{t+k}^{-\sigma}} \quad (9)$$

where $\bar{p}_{H,t} = \frac{\bar{P}_{H,t}}{P_{H,t}}, MC_t^r = \frac{W_t}{P_{H,t}A_t}$, $\Pi_{H,t+k,t}$ is the cumulative domestic inflation rate between periods $t+k$ and t, and $\Pi_{t+k,t}$ is the cumulative CPI inflation rate between periods t+k and t.

The real reset price $\bar{p}_{H,t}$ is

$$\bar{p}_{H,t} = \frac{\epsilon}{\epsilon-1}\frac{\psi_t}{\phi_t} \quad (10)$$

where

$$\psi_t = E_t\sum\nolimits_{k=0}^{\infty}(\beta\theta)^k\Pi_{H,t+k,t}^{\epsilon+1}\Pi_{t+k,t}^{-1}\Pi_{t+k-1,t-1}^{-\zeta\epsilon}MC_{t+k}^{r}\,Y_{t+k}C_{t+k}^{-\sigma} \quad (11)$$

$$\phi_t = E_t\sum\nolimits_{k=0}^{\infty}(\beta\theta)^k\Pi_{H,t+k,t}^{\epsilon}\Pi_{t+k,t}^{-1}\Pi_{t+k-1,t-1}^{-\zeta\epsilon}\,Y_{t+k}C_{t+k}^{-\sigma} \quad (12)$$

These equations can be expressed recursively:

$$\psi_t = MC_t^rY_tC_t^{-\sigma} + \theta\beta E_t(\pi_{H,t+1}^{\epsilon+1}\pi_{t+1}^{-1}\pi_t^{-\zeta\epsilon}\psi_{t+1}) \quad (13)$$

$$\phi_t = Y_tC_t^{-\sigma} + \theta\beta E_t(\pi_{H,t+1}^{\epsilon}\pi_{t+1}^{-1}\pi_t^{-\zeta\epsilon}\phi_{t+1}) \quad (14)$$

Price Dispersion

Following Schmitt-Grohe & Uribe (2007), price dispersion $z_t = \int_0^1\left(\frac{P_{H,j,t}}{P_{H,t}}\right)^{-\epsilon}dj$ can be written as:

$$z_t = (1-\theta)\left(\bar{p}_{H,t}\right)^{-\epsilon} + \theta\left(\left(\pi_{H,t}\right)^{\epsilon}\pi_{t-1}^{-\zeta\epsilon}z_{t-1}\right) \quad (15)$$

2.3. Indices, Assumptions, Definitions, and Identities

In this section, we replicate several indices, assumptions, definitions, and identities as they appear in Gali and Monacelli (2005).

The Domestic Price Index is defined as:

$$P_{H,t} = \left(\int_0^1 P_{H,j,t}^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}} \quad (16)$$

The *Price Index for imported goods from country i in terms of domestic currency* is defined as:

$$P_{i,t} = (\int_0^1 P_{i,j,t}^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}} \tag{17}$$

where $P_{i,j,t}$ is the price of country *i*'s good *j* in terms of domestic currency.

The *Price Index for imported goods in terms of domestic currency* is defined as:

$$P_{F,t} = (\int_0^1 P_{i,t}^{1-\gamma} di)^{\frac{1}{1-\gamma}} \tag{18}$$

where γ is the substitutability between goods produced in different foreign countries.

The *Consumer Price Index* (CPI) is defined as:

$$P_t = [(1-\alpha)P_{H,t}^{1-\upsilon} + \alpha P_{F,t}^{1-\upsilon}]^{\frac{1}{1-\upsilon}} \tag{19}$$

where α is trade openness and υ is the substitutability between domestic and foreign goods.

Bilateral Terms of Trade $S_{i,t}$ are defined as the ratio of the *price index for imported goods from country i in terms of domestic currency* to the *Domestic Price Index*.

$$S_{i,t} = \frac{P_{i,t}}{P_{H,t}} \tag{20}$$

Terms of Trade S_t are defined as the ratio of the *price index for imported goods in terms of domestic currency* to the Domestic Price Index.

$$S_t = \frac{P_{F,t}}{P_{H,t}} = (\int_0^1 S^{1-\gamma}{}_{i,t} di)^{\frac{1}{1-\gamma}} \tag{21}$$

The Law of One Price holds for all individual goods.

$$P_{i,j,t} = \varepsilon_{i,t} P_{i,j,t}^i \tag{22}$$

where $P_{i,j,t}^i$ is the price of country *i*'s good *j* in terms of country *i*'s currency, and $\varepsilon_{i,t}$ is the bilateral nominal exchange rate between country *i* and the home economy.

The Bilateral Real Exchange Rate is the ratio of country *i*'s CPI to the domestic economy's CPI.

$$Q_{i,t} = \frac{\varepsilon_{i,t} P_{i,t}^i}{P_t} \tag{23}$$

where P_t^i is the CPI for country *i*. Note that aggregating bilateral nominal exchange rates over i equals nominal exchange rate e_t (i.e. $e_t = (\int_0^1 (\varepsilon_{i,t})^{1-\upsilon} di)^{\frac{1}{1-\upsilon}}$).

2.4. Monetary Policy Rule

We consider two monetary policy rules:

$$\text{Rule I:} \left(\frac{1+i_t}{1+\bar{\iota}}\right) = \left(\frac{1+i_{t-1}}{1+\bar{\iota}}\right)^{\rho_i} \left(\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t}{y}\right)^{\phi_y}\right)^{1-\rho_i} \exp\left(v_t\right) \tag{24}$$

$$\text{Rule II:} \left(\frac{1+i_t}{1+\bar{\iota}}\right) = \left(\frac{1+i_{t-1}}{1+\bar{\iota}}\right)^{\rho_i} \left(\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t}{y}\right)^{\phi_y} \left(\frac{g_{e,t}}{g_e}\right)^{\phi_e}\right)^{1-\rho_i} \exp\left(v_t\right) \tag{25}$$

where $v_t = \rho_v v_{t-1} + e_{v,t}$ is monetary policy shock where $e_{v,t}$ is the shock innovation, and ϕ_π, ϕ_y and ϕ_e are the coefficients of the monetary policy rule's variables. ρ_v is the persistence of monetary policy shock, and ρ_i is monetary policy inertia parameter. $g_{e,t} \equiv e_t - e_{t-1}$ is the growth rate of nominal exchange rate. y, π, $\bar{\iota}$, and g_e are the steady state values of output, inflation rate, nominal interest rate, and nominal exchange rate growth.

2.5. Risk-Sharing Condition

Following Gali and Monacelli (2005), we obtained the following international risk-sharing condition:

$$E_t\left[\frac{(C_{t+1})}{(C_t)}\right] = E_t\left[\frac{({C^i}_{t+1})}{({C^i}_t)}\left(\frac{Q_{i,t+1}}{Q_{i,t}}\right)^{\frac{1}{\sigma}}\right] \tag{26}$$

where ${C^i}_t$ is the composite consumption index for country *i*. Gali and Monacelli (2005) derive this condition under the assumption of a complete financial market.

$$C_t = \vartheta_{i,t}\, {C^i}_t \left(Q_{i,t}\right)^{\frac{1}{\sigma}} \tag{27}$$

where $\vartheta_{i,t}$ is a pre-selected constant. Without loss of generality, we assume symmetric initial conditions in all small open economies. Thus, $\vartheta_{i,t} = 1$.

2.6. Market-Clearing Condition

The market-clearing condition is the same as in Gali & Monacelli (2005):

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-v} C_t\left[(1-\alpha) + \alpha \int_0^1 \left(S_t^i S_{i,t}\right)^{\gamma - v} Q_{i,t}^{v-\frac{1}{\sigma}}\right] di \tag{28}$$

where S_t^i represents the terms of trade for country *i*.

2.7. Uncovered Interest Rate Parity

Households are allowed to invest in domestic and foreign bonds under a complete international financial market. Optimizing these assets yields the following relation:

$$(1+i_t) = (1+i_t^*)E_t\left(\frac{e_{t+1}}{e_t}\right) \tag{29}$$

This relation is known as the uncovered interest rate parity condition.

3. Data

In our analysis, five seasonally adjusted quarterly time series are employed: GDP y_t, CPI inflation π_t, nominal interest rate i_t, real exchange rate q_t and nominal exchange rate e_t. All data are obtained from the Central Bank of the Republic of Turkey (CBRT). The Industrial Production Index is a proxy for GDP. The nominal interest rate is measured as Interbank Overnight Cash Rate and expressed as an annualized percentage. The rest of the variables are expressed quarterly. To calculate the quarterly CPI inflation rate, we use the quarterly Consumer Price Index. All five series are demeaned and seasonally adjusted. The data cover the period between Quarter II of 2010 and Quarter IV of 2016. The corresponding measurement equations are:

$$\begin{array}{c} 100\Delta(\log(y_t) - \log(mean(y)) \\ 100(\log(\pi_t) - \log(mean(\pi)) \\ 400(\log(i_t) - \log(mean(i)) \\ 100(\log(e_t) - \log(mean(e)) \\ 100(\log(q_t) - \log(mean(q)) \end{array} = \begin{array}{c} \hat{y}_t - \hat{y}_{t-1} \\ \hat{\pi}_t \\ 4\hat{\imath}_t \\ \hat{e}_t \\ \hat{q}_t \end{array}$$

Calibration and Priors

We follow several standard Calibration values derived from relevant literature. β is set to 0.99, γ is set to 1 and υ is set to 1, and ϵ is set to 7.67 as in Alp and Elekdag (2011). We set σ to 3.25 by following Çebi (2015). We assume independent and identically distributed (*i.i.d.)* monetary policy shock and demand shock d_t. We set indexation parameter ζ to 0 to avoid identification problems, and the degree of openness is set to 0.27. The rest of the parameters are econometrically estimated. Table 1 indicates the specifications of the priors. At the onset of estimation, we choose 0.95 as an initial value for ϕ_π. For trend inflation rate, we choose 8% trend inflation rate as mean and 2% trend inflation rate as standard deviation. These values are consistent with the moments of CPI inflation rate in Turkey. For the distribution of trend inflation rate, we follow gamma distribution, and it is consistent with the literature. For distribution of ls, we follow Farmer et al. (2015) and choose the moments of them arbitrarily. We follow Haque et al. (2021) for the distribution and moments of σs. For the type of distribution and moments of θ and φ, we follow Çebi (2015). We choose the same distribution for monetary policy coefficients as those in the literature (Alp and Elekdag, 2011, Zhang and Dai, 2020), but close prior moments. The reason behind this adjustment is to avoid any strong biasedness towards either determinacy or indeterminacy. The rest of the parameters' distributions, prior means, and prior standard deviations are taken from the literature. The reason behind using close prior moments is to guarantee considerable biasedness towards either determinacy or indeterminacy. We set world inflation to 0.

Table 1 Prior Means and Standard Deviations

Parameter Name	Density	Prior Mean	Prior Stand. Dev.
π	Gamma	8	2
θ	Beta	0.50	0.10
ϕ_π	Gamma	1.25	0.50
ϕ_y	Gamma	0.25	0.10
ϕ_e	Gamma	0.05	0.01
φ	Normal	2	0.50
ρ_a	Beta	0.70	0.10
ρ_{y^*}	Beta	0.50	0.10
ρ_i	Beta	0.50	0.20
$\sigma_{u,1}$	Inv-gamma	0.50	0.20
$\sigma_{u,2}$	Inv-gamma	0.50	0.20
$\sigma_{u,3}$	Inv-gamma	0.50	0.20
$\sigma_{u,4}$	Inv-gamma	0.50	0.20
$\sigma_{u,5}$	Inv-gamma	0.50	0.20
l_{11}	Inv-gamma	1	0.50
l_{21}	Normal	0	1
l_{22}	Inv-gamma	1	0.50
l_{31}	Normal	0	1
l_{32}	Normal	0	1
l_{33}	Inv-gamma	1	0.50
l_{41}	Normal	0	1
l_{42}	Normal	0	1
l_{43}	Normal	0	1
l_{44}	Inv-gamma	1	0.50
l_{51}	Normal	0	1
l_{52}	Normal	0	1
l_{53}	Normal	0	1
l_{54}	Normal	0	1
l_{55}	Normal	0	1

4. Estimation Strategy

We first log-linearize the equations of the model around deterministic steady state values[5]. For our model's estimation, we next follow Lubik and Schorfheide (2003,2004) with Farmer et al.'s (2015) approach.

The model can be re-written in linear rational expectation form:

$$o_0\,(\Theta)X_t \;= o_1\,(\Theta)X_{t-1} \;+\; o_2\,(\Theta)m_t \;+ o_3\,(\Theta)\eta_t$$

where $X_t \in R^k$ is a vector of deviations from the means of macroeconomic variables, m_t is an $l \times 1$ vector of exogenous (or fundamental) and mean-zero shocks, and η_t is a p $\times$ 1 vector of rational expectation forecast errors (or endogenous/non-fundamental shocks). $o_0(\Theta), o_1\,(\Theta), o_2\,(\Theta),$ and $o_3\,(\Theta)$ are coefficient matrices. Θ is the vector of the model's parameters. $o_0(\Theta)$ and $o_1\,(\Theta)$ are dimensions of $k \times\ k$ matrices. $o_2(\Theta)$ and $o_3\ (\Theta)$ are dimensions of $k\ \times\ l$ and $k \times\ p$ matrices, respectively.

To calculate the indeterminate set of equilibria, we close the model by converting non-fundamental errors to fundamental errors.

It is assumed that:

[5] See Appendix for the log-linearized equations of the model.

$$E_{t-1}\, m_t \;=\; 0 \; and \; E_{t-1}\eta_t \;=\; 0$$

We define the covariance matrix of exogenous shocks Ω_{mm} as:

$\Omega_{mm} = E_{t-1}(m_t\, m_t')$

We assume that the model has n unstable eigenvalues and p non-fundamental errors. Under some regularity assumptions, there will be $m = p - n$ degrees of indeterminacy. To treat indeterminate models as determinate, m non-fundamental errors are redefined as new fundamental shocks.

We partition the η_t into two pieces: $\eta_{f,t}$ and $\eta_{n,t}$.

$$\underset{kxk}{o_0}\ \underset{}{(\Theta)}\ \underset{kx1}{X_t} = \underset{kxk}{o_1}\ (\Theta)\underset{kx1}{X_{t-1}} + \underset{kxl}{o_2}\ (\Theta)\underset{lx1}{m_t} + (\underset{kxm}{o_{3,f}}\ (\Theta)\ \underset{kxn}{o_{3,n}}\ (\Theta))\begin{pmatrix} \underset{mx1}{\eta_{f,t}} \\ \underset{nx1}{\eta_{n,t}} \end{pmatrix}$$

where $\eta_{f,t}$ is newly-defined fundamental errors and $\eta_{n,t}$ is the remaining non-fundamental errors.

We then re-write the system:

$$\underset{kxk}{o_0}\ (\Theta)\ \underset{kx1}{X_t} = \underset{kxk}{o_1}\ (\Theta)\underset{kx1}{X_{t-1}} + (\underset{kxl}{o_2}\ (\Theta)\underset{kxm}{o_{3,f}}(\Theta))\ \tilde{m}_t + (\underset{kxn}{o_{3,n}}\ (\Theta))\underset{nx1}{\eta_{n,t}}$$

where

$$\tilde{m}_t = \begin{pmatrix} \underset{lx1}{m_t} \\ \underset{mx1}{\eta_{f,t}} \end{pmatrix}$$

We choose newly-defined fundamental shock using CPI inflation:

$$\eta_{f,t} \;=\; \hat{\pi}_t \;-\; E_{t-1}\,\hat{\pi}_t$$

To allow fundamental shocks to co-vary with each other, we define all shocks' processes as:

$$e_{v,t} \;=\; l_{11}u_{v,t}$$

$$e_{a,t} \;=\; l_{21}u_{v,t} + \; l_{22}\, u_{a,t}$$

$$e_{y^*,t} \;=\; l_{31}u_{v,t} + \; l_{32}\, u_{a,t} \;+\; l_{33}\, u_{y^*,t}$$

$$e_{d,t} \;=\; l_{41}u_{v,t} + \; l_{42}\, u_{a,t} \;+\; l_{43}\, u_{y^*,t} \;+\; l_{44}u_{d,t}$$

$$\eta_{f,t} \;=\; l_{51}u_{v,t} + \; l_{52}\, u_{a,t} \;+\; l_{53}\, u_{y^*,t} \;+\; l_{54}u_{d,t} \;+\; l_{55}\, u_{\eta,t}$$

where all ls are coefficients, and $u_{v,t}$, $u_{a,t}$, $u_{y^*,t}$, $u_{d,t}$ and $u_{\eta,t}$ are the shocks' innovations.

Parameter vector Θ is:

$[\pi, \theta, \sigma, \beta, \alpha, \epsilon, \upsilon, \zeta, \gamma, \phi_\pi, \phi_y, \phi_e, \varphi, \rho_d, \rho_v, \rho_a, \rho_y^*, \rho_i, \sigma_{u_1}, \sigma_{u_2}, \sigma_{u_3}, \sigma_{u_4}, \sigma_{u_5}, l_{11}, l_{21}, l_{22}, l_{31}, l_{32}, l_{33}, l_{41}, l_{42}, l_{43}, l_{44}, l_{51}, l_{52}, l_{53}, l_{54}, l_{55}]'$

$$X_t = [\hat{c}_t, \hat{n}_t, \widehat{w}_t^r, \widehat{mc}_t^r, \hat{\pi}_t, d_t, \hat{a}_t, \hat{\imath}_t, \hat{\psi}_t, \hat{\phi}_t, \hat{r}_t, v_t, \hat{\pi}_{H,t}, \hat{s}_t, \hat{y}_t, \hat{e}_t, \hat{q}_t, \hat{\pi}_t^*, \hat{y}_t^*, \hat{z}_t, E_t\hat{\psi}_{t+1}, E_t\hat{\phi}_{t+1}, E_t\,\hat{\pi}_{t+1}, E_t\,\hat{\pi}_{H,t+1}, E_t\,\hat{c}_{t+1}, e_{v,t}, e_{a,t}, e_{d,t}, e_{y^*,t}, \eta_t]'$$

We choose prior means and prior standard deviations, which lead to a probability of determinacy of 0.54 and 0.58 for the first and second rule, respectively. This is to avoid bias towards either determinacy or indeterminacy. In our estimation, we use the Bayesian method with 6 chains and 1,500,000 draws from each chain.

5. Results of Empirical Analysis

In this section, we analyze the model using the CPI inflation rate, nominal interest rate, nominal exchange rate, and output growth.

Table 2 Posterior Means and Standard Deviations

	Rule I		Rule II	
	Mean	Standard Dev.	Mean	Standard Dev.
π	9.54	2.20	9.36	2.17
θ	0.73	0.04	0.73	0.04
ϕ_π	0.78	0.35	0.77	0.36
ϕ_y	0.31	0.09	0.33	0.10
ϕ_e	-	-	0.05	0.01
φ	1.59	0.55	1.59	0.56
ρ_a	0.61	0.11	0.62	0.11
ρ_{y^*}	0.32	0.08	0.32	0.07
ρ_i	0.50	0.17	0.55	0.15
$\sigma_{u,1}$	0.19	0.02	0.19	0.02
$\sigma_{u,2}$	0.31	0.07	0.31	0.07
$\sigma_{u,3}$	0.24	0.03	0.24	0.03
$\sigma_{u,4}$	0.19	0.02	0.19	0.02
$\sigma_{u,5}$	0.45	0.11	0.45	0.11
l_{11}	0.33	0.04	0.33	0.04
l_{21}	-0.05	0.79	-0.27	0.80
l_{22}	0.69	0.13	0.69	0.14
l_{31}	0.22	0.22	0.25	0.22
l_{32}	-0.02	0.08	-0.04	0.07
l_{33}	0.44	0.06	0.44	0.06
l_{41}	-0.68	0.25	-0.63	0.26
l_{42}	-0.02	0.07	-0.04	0.07
l_{43}	0.35	0.08	0.36	0.08
l_{44}	0.33	0.04	0.33	0.04
l_{51}	-1.12	0.26	-1.10	0.27
l_{52}	-0.03	0.05	-0.03	0.04
l_{53}	-0.20	0.05	- 0.18	0.05
l_{54}	-0.15	0.16	-0.14	0.17
l_{55}	-0.00	0.02	0.00	0.02

5.1. Parameter Estimates

Table 2 presents posterior means and standard deviations of the parameters under two monetary policy rules. The values of individual parameters are near each other under both rules. As seen in Table 2, the policy responses to inflation rate and output are weak. Gürkaynak et al. (2015) empirically support our findings. The long-run inflation rate is less than 10%. According to our findings, domestic price levels are quite rigid. Moreover, the inertial parameter of the Taylor rule is moderate. On the other hand, long-run inflation rate volatility is high while we see low standard deviations in the coefficients of inflation rate and output.

5.2. Testing for Indeterminacy

Using posterior means and standard deviations, we calculate the posterior probability of determinacy and indeterminacy of the model based on 1,000 draws from the posterior parameter vector under alternative monetary policies.

Table 3: Probabilities of Determinacy and Indeterminacy

Rule	Determinacy	Indeterminacy
1	0.02	0.98
2	0.02	0.98

Table 3 indicates that the probability of indeterminacy is close to 1 under both rules. The economy is highly likely to be indeterminate as monetary policy rules are not strong enough to remove sunspot fluctuations and varying propagation mechanism of shocks in macroeconomic variables. Thus, macroeconomic variables may fluctuate arbitrarily.

6. What leads to indeterminacy?

In this section, we discuss possible causes of indeterminacy.

6.a. Long-run Inflation Rate

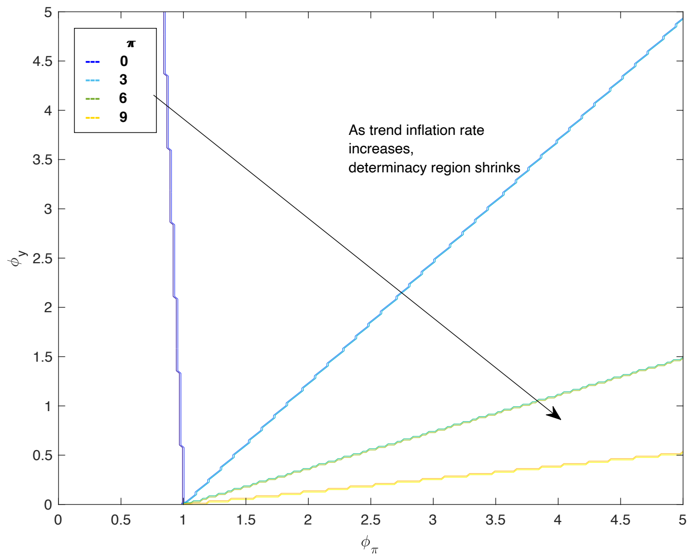

Figure 1: How determinacy region responds to increases in long-run inflation rate

In this subsection, we evaluate how increased long-run inflation affects the model's determinacy properties.[6] Figure 1 indicates that the determinacy region shrinks at higher long-run

[6] In this subsection, we set $\alpha = 0.30, \theta = 0.75, \sigma = 3, \gamma = \upsilon = 1, \zeta = 0, \varphi = 2, and\ \epsilon = 7.67$.

inflation rates. Our findings are in line with Ascari and Sbordone (2014) and Ascari and Ropele (2009). This dynamic results from the non-linear relation between long-run inflation and output. As the long-run inflation rate increases, output decreases (Ascari and Sbordone, 2014). Thus, the determinacy region shrinks more at higher long-run inflation rates.

6.b. Trade Openness

In this subsection, we discuss how increased trade openness affects the model's determinacy properties at positive long-run inflation rates.[7] Figure 2 indicates that higher trade openness leads the determinacy region to shrink more at positive long-run inflation rates when compared to the closed economy. Our findings are in line with Kara et al. (2021). The movement arises from the long-run relation between output and inflation rate. Higher trade openness weakens this relation (Kara et. al, 2021). Therefore, the determinacy region shrinks more compared to a closed economy.

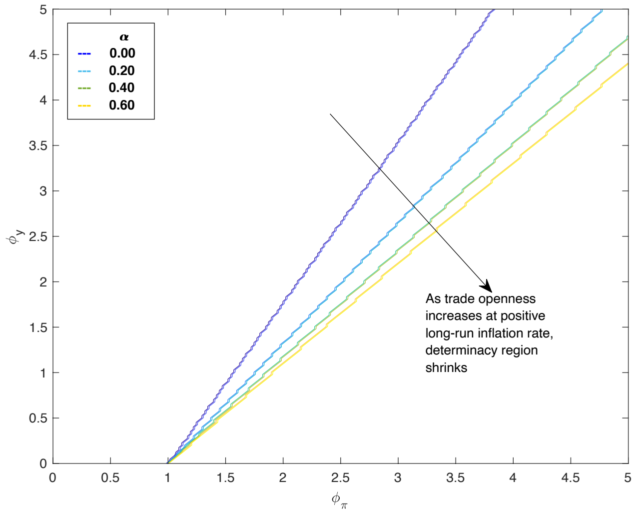

Figure 2: How the determinacy region responds to increase in trade openness

6.c. Calvo Parameter

In this subsection, we analyze how changes in the Calvo parameter affect the model's determinacy properties.[8] Figure 3 indicates that higher Calvo parameters shrink the determinacy region at positive long-run inflation rates. A higher Calvo parameter makes the NKPC more forward-looking, leading to a smaller determinacy region.

[7] We set $\pi = 3, \theta = 0.75, \sigma = 3, \gamma = \upsilon = 1, \zeta = 0, \varphi = 2$ and $\epsilon = 7.67$ in this subsection.
[8] We set $\pi = 3, \alpha = 0.30, \sigma = 3, \gamma = \upsilon = 1, \zeta = 0, \varphi = 2$ and $\epsilon = 7.67$ in this subsection.

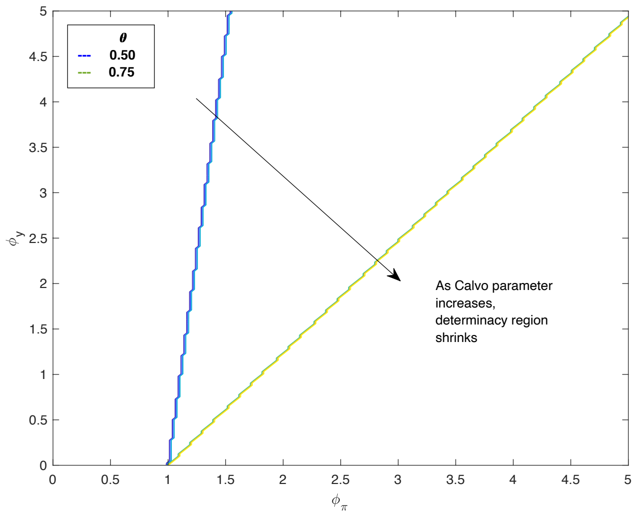

Figure 3: How the determinacy region responds to change in the Calvo parameter

A shrinking determinacy region implies that the Central Bank should respond strongly to changes in inflation rate and weakly to output in order to keep the economy determinate. Otherwise, macroeconomic variables are not uniquely determined. Assuming that nominal interest rates remain constant, any increase in inflation expectations leads to a decrease in real interest rate and leads to increased output through the Euler Equation. Thereafter, the inflation rate increases through the NKPC. As long as the nominal interest rate remains constant, this dynamic repeats and results in self-fulfilling fluctuations in macroeconomic variables.

7. Robustness

In this section, we perform robustness analysis using the CPI inflation rate, nominal interest rate, real exchange rate, and growth of output.

Table 4: Probabilities of Determinacy and Indeterminacy

Rule	Determinacy	Indeterminacy
1	0.02	0.98
2	0.01	0.99

Table 4 indicates that, under both rules, the probabilities of indeterminacy is considerably greater than the ones of determinacy. The economy is thus highly likely to be indeterminate. These findings are in line with benchmark cases.

8. Conclusion

In this paper, we evaluate the stability of the Turkish economy in the early 2010s using a small open economy New-Keynesian model featuring a positive long-run inflation rate and inflation rate indexation under alternative monetary policy rules. We first estimate the model's empirical results under alternative monetary policy rules. The model's empirically results indicate that long-run inflation rate is high and the Calvo parameter is moderate despite the Central Bank's weak response to inflation rate and output. We found that the economy has a high likelihood towards indeterminacy under all rules. High long-run inflation and high Calvo parameters are the driving forces of indeterminacy in the economy, although trade openness aggravates the effects of long-run inflation on the determinacy region.

References

Alp, H. and Elekdag, S. (2011). "The Role of Monetary Policy in Turkey during the Global Financial Crisis," *Central Bank of the Republic of Turkey*, WP No: 1110.

Araújo, E. (2014). "Determinacy and Learnability of Equilibrium in a Small Open Economy with Sticky Wages and Prices," *Central Bank of Brazil*, WP No: 376.

Ascari, G. and Sbordone, A. M. (2014). "The Macroeconomics of Trend Inflation," *Journal of Economic Literature*, 52(3):679-739.

Ascari, G. and Ropele, T. (2009). "Trend Inflation, Taylor Principle and Indeterminacy," *Journal of Money, Credit and Banking*, 41(7): 1557-1584.

Calvo, G.A., (1983). "Staggered Prices in A Utility-Maximizing Framework." *Journal of Monetary Economics*, 12(3): 383-398.

Çebi, C., (2011). "The Interaction Between Monetary and Fiscal Policies in Turkey: An Estimated New Keynesian DSGE Model," *Economic Modelling*, 29(4): 1258-1267.

De Fiore, F. and Liu, Z. (2005). "Does Trade Openness Matter for Aggregate Instability?," *Journal of Economic Dynamics and Control*, 29(7): 1165-1192.

Farmer, R.E.A., Khramov, V. and Nicolo, G. (2015). "Solving and Estimating Indeterminate DSGE Models," *Journal of Economic Dynamics and Control*, 54:17-36.

Gali, J. and Monacelli, T. (2005). "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72(3): 707-734.

Gürkaynak, R.S., Kantur, Z. Taş, M.A. and Yıldırım, S. (2015). "Monetary Policy in Turkey After Central Bank Independence," *Iktisat Isletme ve Finans*, 30(356), 09-38.

Haque, Q., Groshenny, N., and Weder, M. (2021). "Do We Really Know That U.S. Monetary Policy Was Destabilizing in the 1970s?," *European Economic Review*, 131: 103615.

Kara, E., Yilmaz, Y. O., and Tunc, G. I., "Indeterminacy, Trade Openness, Trend Inflation and Heterogeneity in Price Stickiness," Available at SSRN 4015222.

Llosa, L. and Tuesta, V. (2008). "Determinacy and Learnability of Monetary Policy Rules in Small Open Economies." *Journal of Money, Credit and Banking*, 40(5): 1033-1063.

Lubik, T. and Schorfheide, F. (2003). "Computing Sunspot Equilibria in Linear Rational Expectations Models," *Journal of Economic Dynamics and Control*, 28(2): 273-285.

Lubik, T.A. and Schorfheide,F. (2004). "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review*, 94(1): 190-217.

Özatay, F. (2011). "Merkez Bankası'nın Yeni Para Politikası:İki Hedef-Üç Ara Hedef-Üç Araç (New Monetary Policy of the Central Bank: Two Targets-Three Intermediate Targets-Three Tools)," *İktisat İşletme ve Finans*, 26 (302): 27-43.

Schmitt-Grohe, S. and Uribe, M. (2007). "Monetary Policy Under Inflation Targeting," in K. Schmidt-Hebbel and R. Mishkin (ed.), *Optimal Inflation Stabilization in a Medium-scale Macroeconomic Model*: 125-186. Central Bank of Chile.

Woodford, M. (2003). *Interest and Prices: Foundations of A Theory of Monetary Policy*. Princeton: Princeton University Press.

Zhang, B. and Dai, W. (2020). "Trend Inflation and Macroeconomic Stability in A Small Open Economy," *Economic Modelling*, 91: 769-778.

Appendix

In this section, we display a log-linear approximation of the model equations. The hat variable implies a log deviation of the variable from its steady state.

$$E_t(\hat{c}_{t+1}) - (\hat{c}_t) = \frac{1}{\sigma} E_t(\hat{\imath}_t - \hat{\pi}_{t+1}) - d_t \tag{A.1}$$

$$\sigma\hat{c}_t + \varphi\hat{n}_t = \widehat{w}^r{}_t \tag{A.2}$$

$$\hat{n}_t = \hat{y}_t - \hat{a}_t + \hat{z}_t \tag{A.3}$$

$$\widehat{mc}^r{}_t = \widehat{w}^r{}_t + \alpha\hat{s}_t - \hat{a}_t \tag{A.4}$$

$$\widehat{\tilde{p}_{H,t}} = \frac{\theta\pi^{(\epsilon-1)(1-\zeta)}}{1-\theta\pi^{(\epsilon-1)(1-\zeta)}} \left(\hat{\pi}_{H,t} - \zeta\hat{\pi}_{t-1}\right) \tag{A.5}$$

$$\widehat{\tilde{p}_{H,t}} = \hat{\psi}_t - \hat{\phi}_t \tag{A.6}$$

$$\begin{gathered}\hat{\psi}_t = (1-\theta\beta\pi^{\epsilon(1-\zeta)})(\widehat{mc}^r{}_t + \hat{y}_t - \sigma(\hat{c}_t)) + \\ \theta\beta\pi^{\epsilon(1-\zeta)}E_t(-\epsilon\zeta\hat{\pi}_t - \hat{\pi}_{t+1} + (\epsilon+1)\hat{\pi}_{H,t+1} + \hat{\psi}_{t+1})\end{gathered} \tag{A.7}$$

$$\begin{gathered}\hat{\phi}_t = \left(1-\theta\beta\pi^{(\epsilon-1)(1-\zeta)}\right)(\hat{y}_t - \sigma\hat{c}_t) + \\ \theta\beta\pi^{(\epsilon-1)(1-\zeta)}E_t((-\epsilon)\zeta\hat{\pi}_t - \hat{\pi}_{t+1} + \epsilon\hat{\pi}_{H,t+1} + \hat{\phi}_{t+1}))\end{gathered} \tag{A.8}$$

$$\begin{gathered}\hat{z}_t = -\epsilon\left(1-\theta\pi^{\epsilon(1-\zeta)}\right)\widehat{\tilde{p}_{H,t}} + \\ \theta\pi^{\epsilon(1-\zeta)}(-\epsilon\zeta\hat{\pi}_{t-1} + \epsilon\hat{\pi}_{H,t} + \hat{z}_{t-1})\end{gathered} \tag{A.9}$$

$$\hat{\pi}_t = \hat{\pi}_{H,t} + \alpha\Delta\hat{s}_t \tag{A.10}$$

$$\hat{q}_t = (1-\alpha)\hat{s}_t \tag{A.11}$$

$$\hat{y}_t = \hat{c}_t + \alpha\gamma\hat{s}_t + \alpha(\upsilon - \frac{1}{\sigma})\,\hat{q}_t \tag{A.12}$$

$$\hat{y}_t = \hat{c}_t + \frac{\alpha\omega}{(1-\alpha)\sigma}\hat{q}_t \tag{A.13}$$

where

$$\omega = \sigma\gamma + (1-\alpha)(\sigma\upsilon - 1)$$

$$\hat{y}^*{}_t = \hat{c}^*{}_t \tag{A.14}$$

where
$\hat{y}_t^* = \rho_{y^*}\,\hat{y}^*{}_{t-1} + e_{y^*,t}$,
ρ_{y^*} is the persistence of world output shock
and $e_{y^*,t}$ is the shock innovation.

$$\hat{c}_t = \hat{c}^*{}_t + \frac{1}{\sigma}\hat{q}_t \tag{A.15}$$

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1-\rho_i)\big(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \phi_e(\hat{e}_t - \hat{e}_{t-1})\big) + v_t \tag{A.16}$$

$$\hat{\imath}_t = \hat{\imath}^*{}_t + E_t(\hat{e}_{t+1} - \hat{e}_t) \tag{A.17}$$