A Note on Labor-Search Models*

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Abstract

In this paper, I revisit the "Shimer puzzle" by comparing business-cycle properties of three prominent labor-search models. Providing a standard framework for contrasting the three models, I first quantify the sensitivity of Shimer (2005)'s relative labor market tightness volatility to business cycle smoothing parametrization. Second, I document that Hagedorn and Manovskii (2008)'s proposed solution to the Shimer puzzle depends considerably on whether the value of nonmarket activity responds to business cycle fluctuations. Third, I report that when specified in the form of a decreasing-returns-to-scale functional form, the incorporation of decreasing average vacancy posting costs by Pissarides (2009) cannot generate decent business-cycle volatilities in the absence of further wage stickiness. Overall, my findings point to the particular need for further exploring the degree of co-movement between wage rate and nonmarket activity in labor-search modelling attempts.

Keywords: Shimer's puzzle, Schmitt-Grohé and Uribe algorithm, labor market tightness.
JEL Classification: E24, E30, E32, J3.

Emek-Arama Modelleri Üzerine Bir Not

Özet

Bu makale, üç farklı emek-arama model spesifikasyonunun iş döngüsü performanslarını karşılaştırarak "Shimer bulmacasını" yeniden ziyaret etmektedir. Üç model için standart bir çerçeve oluşturduktan sonra ilk olarak Shimer'in (2005)'ın işgücü piyasasındaki sıkılığın verimlilik oranına olan düşük oynaklık sonucunun iş döngüsü yumuşatma

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parametresine duyarlılığı ölçülmüştür. İkinci olarak Hagedorn ve Manovskii (2008)'nin Shimer bulmacasına önerdiği çözümün pazardışı etkinliğin iş döngüsü dalgalanmalarına bağlılığı ortaya konmuştur. Üçüncü olarak Pissarides (2009) tarafından öne sürülen azalan ortalama iş ilanı masraf fikrinin azalan-getiri-dönüş fonksiyonel formu ile formüle edilmesinin yapışkan ücretler noksanlığında iyi iş döngüsü oynaklıkları sağlayamadığı bildirilmektedir. Genel olarak, tüm bu bulgular ücret oranı ile pazardışı aktivitenin beraber hareketlilik oranının daha iyi emek-arama modelleme için araştırılması ihtiyacını ortaya koymaktadır.

Anahtar Kelimeler: Shimer bulmacası, Schmitt-Grohé ve Uribe algoritması, iş piyasası sıkılığı
Jel Sınıflaması: E24, E30, E32, J3.

Over the recent decades, labor market dynamics have become an issue of high priority to both policy-makers and researchers. The fact that frictionless standard real business cycle models fail to generate realistic labor market dynamics has motivated numerous studies to find alternative ways to explain observed irregularities. [(1)] These attempts have contributed to the emergence of three major sources of labor market imperfections: i) implicit contract theory relates observed wage stickiness to risk- aversion of workers and employers, [(2)] ii) efficiency wage theory by Shapiro and Stiglitz (1984) argues for economic efficiency as a result of unemployment due to its disciplinary device function, and iii) labor-search theory by Mortensen and Pissarides (1994) attempts to endogenize labor market frictions by the use of job creation and job destruction in a partial equilibrium design. Among the three branches, labor-search models have gained particular popularity in modern economics for the study of labor market imperfections, and therefore have been incorporated into several general equilibrium models.[(3)]

One unique result by partial and general equilibrium models featuring labor-search has been particularly controversial: the unemployment and vacancy volatilities generated by labor-search models are an order of magnitude lower than those in the data. In the literature, this phenomenon has been coined as the "Shimer's puzzle," and it has motivated alternative ways to tackle the mismatch. In one of these attempts, Hagedorn and Manovskii (2008) argue that because of similar grounds as in Hansen (1985), and the fact that partial labor-search models can be regarded as a linear approximation to standard non-linear real business cycle models, nonmarket activity (i.e. unemployment benefit/home production) needs to be set close to wage rate. [(4)] Accordingly, Hagedorn and Manovskii (2008) show that when nonmarket activity is formulated as a parameter, the value of which is calibrated close to steady-state wage rate, volatilities of unemploy-

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(1) Among others, see Shimer (2010).

(2) See Rosen (1985) for an elaborate discussion on the implicit contract theory.

(3) Among others, see Merz (1995) and Andolfatto (1996).

(4) Specifically, Hagedorn and Manovskii (2008) refers to Hansen (1985)'s indivisible labor argument for the study of real business cycle models, in which households are a priori indifferent between working and not working.

ment and vacancies fit considerably better with the data. In another attempt addressing the Shimer puzzle, Pissarides (2009) shows that imposing only exogenous wage stickiness would not satisfactorily resolve the puzzle. Instead of standard *proportional* vacation posting costs, he proposes a fixed-cost-embedded decreasing-vacancy- cost formulation, which he argues for a better match with the data.

In this paper, I study the behavior of macroeconomic variables in different labor-search model specifications with a comparative perspective. Specifically, extending on the canonical labor-search model a` la Shimer (2005), I revisit the "Shimer puzzle" by contrasting business-cycle performances of three prominent labor-search models.

First, I quantify the sensitivity of Shimer (2005)'s relative labor market tightness volatility to business-cycle smoothing parametrization, and document that the use of standard quarterly smoothing parameter reduces labor market tightness volatility by as much as one-fourth.[(5)]

Second, I study the Hagedorn and Manovskii (2008) economy, which features a nonmarket activity value close to that of wage rate. Throughout my analyses, I formulate the value of nonmarket activity in two different approaches. First, I specify nonmarket activity as a parameter, the unique value of which is determined at the deterministic steady-state as a *ratio* of the wage rate and remains constant regardless of labor productivity shocks, as in Hagedorn and Manovskii (2008). Second, instead of fixing the value nonmarket activity to a constant, I *endogenize* the value of nonmarket activity so that the ratio of nonmarket activity to wage rate remains constant not only at the steady-state, but also throughout business-cycle fluctuations. In other words, the second formulation suggests that the value of nonmarket activity that mainly addresses home production or unemployment benefit does not necessarily have to be orthogonal to labor productivity shocks, and responds to the developments in the economy. [(6)] I show that the formulation of nonmarket activity has first-order implications: when nonmarket activity is formulated as a parameter, the standard Hagedorn and Manovskii (2008) result holds, as the relative volatility of labor market tightness is amplified by as much as an order of magnitude. However, when nonmarket activity is endogenized as described, the otherwise standard Hagedorn and Manovskii model generates a relative labor market tightness volatility figure similar to that of the Shimer model. These findings suggest that co-movement of nonmarket activity and wage rate throughout business-cycles is to be explored for a better understanding of the Shimer's puzzle and for the formulating of consequent model-based solutions.

Third, following Pissarides (2009), I modify the standard labor-search model by introducing decreasing vacancy posting costs, instead of proportional ones. In doing so, instead of imposing fixed costs, I employ a decreasing-returns-to-scale vacancy posting cost function so as to formulate decreasing average cost of posting vacancies.

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(5) Specifically, I refer to the finding that the use of Hodrick and Prescott smoothing parameter of $\lambda = 1600$ for the-quarterly frequency, instead of $\lambda = 105$ by Shimer (2005) reduces relative volatility of labor market tightness by 24%.

(6) Throughout the paper, I refer to the two specifications as labor-search models with i) exogenous χ and ii) endogenous χ, respectively, where χ refers to nonmarket activity, as discussed in detail in the Model Environment section.

I report that decreasing average vacancy postings costs proposal by Pissarides (2009) cannot generate decent business-cycle predictions, when formulated as a decreasing-returns-to-scale functional form.

Overall, by highlighting the strengths and shortcomings of canonical labor-search models in a comparative perspective, this paper intends to contribute to a better understanding of tackling the Shimer's puzzle.

The outline of the rest of the paper is as follows: in the Model Environment section, I describe the details of my extensions to the canonical labor search-model a` la Shimer (2005), in Results section, I describe my solution algorithm and present my results, and in the Conclusions section, I present my discussions and conclusions.

Model Environment

Households

There is an infinitely-lived representative household, who has an "infinite" number of family members. The size of the infinite-sized household is normalized to measure one. All household members are assumed to be participating in the labor force. The representative household accordingly maximizes:

$$\max_{\{c_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \widetilde{u}(c_t)$$

subject to

$$c_t = w_t n_t^h + u_t \chi + d_t$$

and

$$n_{t+1}^h = (1-\delta)(n_t^h + u_t p(\theta_t))$$

where $\mathbb{E}_0$ refers to the expectations operator at time t=0, $\widetilde{u}(\cdot)$ refers to the utility function, which is assumed to take the natural logarithm form, β refers to the discount factor, c_t refers to consumption at t, w_t refers to the real wage rate, n_t^h refers to the measure of household members who work and u_t refers to the measure of household members do not work, χ (or χ_t) refers to nonmarket activity, i.e. unemployment benefit/home production, d_t refers to the lump-sum flow of profits due to the household's ownership of the representative firm, δ refers to the time-invariant exogenous job separation rate, θ_t refers to labor market tightness defined per unemployed vacancy, i.e. $(\frac{v_t}{u_t})$, and $p(\theta_t)$ refers to the job finding rate.[7]

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[7] In a general equilibrium model environment, unemployment benefits would not appear in the economy-wide re- source constraint, simply because it is only a form of transfer of resources from one party to the next. In the canonical labor-search model a la Shimer (2005), χ shows up in the resource constraint, hence interpreting χ as home production is a better way of making sense of the model.

The household maximization problem yields:

$$1 = R_t \, \mathbb{E}_t \, \frac{\beta \widetilde{u}'(c_{t+1})}{\widetilde{u}'(c_t)}$$

where R_t denotes the gross return rate of the zero-net-supply risk-free asset. Consequently, the stochastic discount rate can be expressed as:

$$\Xi_{t+1|t} \equiv \frac{\beta \widetilde{u}'(c_{t+1})}{\widetilde{u}'(c_t)}$$

Firms

There is a representative "large" firm, which faces the following profit-maximization problem:

$$\max_{n^f_{t+1}, v_t} \; \mathbb{E}_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left(y_t - w_t n^f_t - \psi v_t \right)$$

subject to

$$n^f_{t+1} = (1-\delta)(n^f_t + v_t q(\theta_t))$$

and

$$y_t = e^{z_t} n_t$$

where $\Xi_{t+1|t} \equiv \frac{\beta \widetilde{u}'(c_{t+1})}{\widetilde{u}'(c_t)}$ refers to the stochastic discount factor, y_t refers to the firm's output, n_t refers to the measure of employed workers in production, ψ refers to the cost of posting vacancies, v_t refers to the number of posted job vacancies, and z_t refers to the labor productivity technology shock. Further, the law of motion for the technology shock is an autoregressive process of order one, which can be described formally as follows:

$$z_{t+1} = (1-\rho_z)\overline{z} + \rho_z z_t + \varepsilon^z_{t+1}$$

where ε^z is normally distributed with $\varepsilon^z \sim N(0, \sigma_z)$.
The optimality condition by the firm yields:

$$\frac{\psi}{q(\theta_t)} = (1-\delta) \, \mathbb{E}_t \, \Xi_{t+1|t} \left(e^{z_{t+1}} - w_{t+1} + \frac{\psi}{q(\theta_{t+1})} \right)$$

which implies that the firm's cost of posting vacancies needs to be equal to the discounted expected profit by filling the vacancy.

Wage Bargaining

Each period, real wage results from a bargaining process between the firm and the worker, and a single wage rate is determined by Nash bargaining. [8] The equilibrium wage solves the following Nash bargaining problem:

$$\max_{w_t} \left(W_t - U_t\right)^{\nu} \left(J_t - V_t\right)^{1-\nu}$$

where $\nu \in (0,1)$ refers to the worker's, and $1-\nu$ refers to the employer's bargaining power. The household's asset value of getting employed, denoted as W_t, follows:

$$W_t = w_t + \mathbb{E}_t \left[\Xi_{t+1|t} \left((1-\delta)W_{t+1} - \delta U_{t+1}\right)\right]$$

The household's asset value of remaining unemployed, denoted as U_t requires:

$$U_t = \chi + \mathbb{E}_t \left[\Xi_{t+1|t} \left(p(\theta_t)(1-\delta)W_{t+1} - (1-p(\theta_t))(1-\delta)U_{t+1}\right)\right]$$

The firm's asset value to fill a vacant job, denoted as J_t, follows:

$$J_t = e^{z_t} - w_t + E_t \left[\Xi_{t+1|t} \left((1-\delta)J_{t+1}\right)\right]$$

Note that Equation (10) and (14) jointly imply that

$$J_t = e^{z_t} - w_t + \frac{\psi}{q(\theta_t)}$$

needs to hold, as well.

Lastly, V_t denotes the asset value of posting a vacancy for a job, which is zero in equilibrium. The solution to the Nash-bargaining problem yields the following surplus-sharing rule:

$$(1-\nu)(W_t - U_t) = \nu J_t$$

Plugging Equations (12), (13), and (14) and (15) into the surplus-sharing rule (16), wage rate follows:

$$w_t = \nu e^{z_t} + (1-\nu)\chi + \nu\psi\theta_t$$

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(8) Under some assumptions, Nash bargaining is known to correspond to the alternating bargaining game a la Rubinstein (1982), hence another way to interpret the determination of wages could be the alternating offers between the worker and the employer in a continuous-time setup.

Equilibrium

The competitive equilibrium for the model economy is a list of sequences: $\{c_t, n_t^h, n_t^f, u_t, v_t, R_t, q(\theta_t), p(\theta_t), w_t, \chi_t{}^9\}_{t=0}^{\infty}$ for the given parameters and the exogenous process z_t, which satisfy the below set of equations (18) (27):
The household's optimal consumption-saving decision rule requires: (9)

$$1 = R_t E_t \frac{\beta \widetilde{u}'(c_{t+1})}{\widetilde{u}'(c_t)}$$

The firm's optimal job-creation and vacancy-posting condition implies: (10)

$$\frac{\psi}{q(\theta_t)} = (1-\delta) E_t \Xi_{t+1|t} \left(e^{z_{t+1}} - w_{t+1} + \frac{\psi}{q(\theta_{t+1})} \right)$$

where the stochastic discount factor follows $\Xi_{t+1|t} \equiv \frac{\beta \widetilde{u}'(c_{t+1})}{\widetilde{u}'(c_t)}$.

Real wage rate solves the Nash-bargaining problem:

$$w_t = \nu e^{z_t} + (1-\nu)\chi + \nu\psi\theta_t$$

Aggregate employment follows its law of motion:

$$n_{t+1} = (1-\delta)(n_t + m(u_t, v_t))$$

The job matching identity holds:

$$m(u_t, v_t) = \kappa u_t^{\xi} v_t^{1-\xi} = p(\theta_t) u_t = q(\theta_t) v_t$$

where $m(\,)$ is a constant-returns-to-scale (CRTS) matching technology function, $q(\,)$. is the rate at $p(.)$ which vacant jobs are filled, and $p(.)$ is the rate at which unemployed individuals get employed.
Labor market clears:

$$n_t^f = n_t^h = n_t$$

Each member of the household participates in the labor pool, either as employed or unemployed:

$$n_t + u_t = 1$$

(9) As discussed in the previous section, when nonmarket activity χ is specified as a *parameter* so that its value is time-invariant and does not respond to labor productivity shocks, it is added to the parameter set instead of the list of sequences.

(10) This condition is modified for the Pissarides model, which is discussed in more detail in the following section.

The law of motion for labor factor productivity follows:

$$z_{t+1} = (1-\rho_z)\overline{z} + \rho_z z_t + \varepsilon^z_{t+1} \quad \text{with} \quad \varepsilon^z_t \sim N(0, \sigma_z).$$

Economy-wide aggregate resource constraint holds:

$$y_t = c_t + \psi v_t - u_t \chi$$

Final output is produced only by the employed workers, and labor productivity is governed by the specified stochastic process: [11]

$$y_t = e^{z_t} n_t$$

Results

Parametrization and Model Specifications

In order to study the business-cycle properties of three competing labor-search models in a comparative approach, I perform three set of specifications as discussed, which I refer hereafter as the Shimer, Hagedorn and Manovskii and Pissarides models. By the Shimer model, I refer to the canonical labor search model a la Shimer (2005), by the Hagedorn and Manovskii model I refer to a modified labor-search model with relatively higher nonmarket activity and lower bargaining power of workers, and by the Pissarides model, I refer to an environment where the cost of posting vacancies is not proportional, but a decreasing function of posted vacancies.

Table 1

Parameter Values of the Three Models

	Shimer Model	**Hagedorn and Manovskii Model**	**Pissarides Model**
β	0.99	0.99	0.99
δ	0.10	0.10	0.10
ξ	0.40	0.40	0.40
κ	0.50	0.50	0.50
ν	0.40	0.052	0.40
ψ	0.25	0.25	$\psi = 0.6019 \times v$ = 0.25
ρ_z	0.95	0.95	0.95
σ_z	0.007	0.007	0.007

β denotes the discount factor, δ denotes the exogenous job separation rate, ξ denotes the exponent of unemployment in the matchin function, κ denotes the productivity rate of matching, ν denotes the worker's bargaining power in Nash bargaining, ψ denotes the cost of posting vacancies, ρz denotes the AR(1) coefficient and σz denotes the standard deviation of the total factor productivity shock.

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(11) Note that this condition is implied by the joint flow budget constraint of the household and the firm. The intuition for this resource constraint is that the total production of real goods in this economy is composed of the firm's production and home production, that is, $y_t + u_t \chi$, out of which the household consumes and the firm covers its real vacancy-posting costs.

(12) Note that when the functional form of the vacancy-posting cost follows Equation (28), the optimal vacancy posting condition for the firm needs to be modified as:

$$\frac{\zeta \iota v_t^{\iota}}{q(\theta_t)} = (1-\delta)\,\mathbb{E}_t\,\Xi_{t+1|t}\left(e^{z_{t+1}} - w_{t+1} + \frac{\zeta \iota v_{t+1}^{\iota}}{q(\theta_{t+1})}\right)$$

I display the baseline parameter values for the three models in Table 1. The parameter values in the Shimer and Pissarides models are identical, except for a minor formulation difference: while the cost of posting vacancies, ψ, is proportional in the Shimer model, I first assume that it takes a decreasing functional form of over posted vacancies in the Pissarides model as follows:

$$\psi(v) = \zeta v^{\iota}$$

Next, while I feed the cost of posting vacancies, ψ, as a constant parameter to the former model, I calibrate the *steady-state value of costing vacancies,* $\overline{\psi} = 0.6019\, \overline{v}^{0.7} = 0.25$ in the latter model so that the equilibrium value of cost of posting vacancies of the two models coincide. (13), (14)

The parameter values of the Hagedorn and Manovskii model differ from the two other models in two dimensions: first, following Hagedorn and Manovskii (2008), I set the bargaining power parameter of the worker in the Hagedorn and Manovskii model, ν, to 0.052, whereas this value is set to 0.40 in the two other models. Second, again following Hagedorn and Manovskii (2008), I set the nonmarket activity parameter value (the ratio of unemployment benefit/home production to steady-state wage rate) in the Hagedorn and Manovskii model to $\chi = 0.95$, whereas this value is set to $\chi = 0.40$ in the two other models. (15)

Model Results

I display the resultant steady-states of the three competing models in Table 2. I report that, as in- tended, steady-state values of the three models are quantitatively similar. The only exception to this similarity is due to nonmarket activity: because of the different parametrization of the Hagedorn and Manovskii model, the steady-state value of nonmarket activity is noticeably higher in the Hagedorn and Manovskii model than those of the two other models. (16)

(13) In doing so, since there is no earlier study that could shed light to this parametrization, I set the exponent of vacancy posting costs to $0 \in (0, 1)$ so that the function displays decreasing returns to scale, and then calibrate the multiplier before $v0.7$ to 0.6019, so that its steady-state value is equal to 0.25, as in the Shimer model. I also re- port the consequent findings with a more concave function cost function in Table A.9-Table A.11. Results with other concavity parametrization are available upon request, and the associated MATLAB codes can be downloaded at http://www.econ.boun.edu.tr/torul/nols.zip.

(14) As in the case of lump-sum transfer of profits, fixed vacancy costs do not alter optimal decision rule of the firm. Fixed-costs, by design, imply decreasing average costs, and the most reasonable way of incorporating Pissarides (2009) argument while also enabling the firm to internalize the costs could be the introduction of a decreasing-returns-to-scale cost function, which affects the firm's optimal decision rule, and thus generates implications of interest by Pissarides (2009)'s claim. For the firm to alter its decision rules due to fixed-costs, one could computationally change the solution algorithm, e.g. rely on a global approximation technique via value or policy function iteration. However, that approach would reduce the comparability of competing models solved via local approximation techniques.

(15) Results of Hagedorn and Manovskii model with $\chi = 0.40$ are displayed in Table A.6, Table A.7 and Table A.8.

(16) As displayed in Table A.6, Table A.7 and Table A.8, in the presence of high nonmarket activity rates (χ =0.95), when bargaining power of workers increase from 0.052 to 0.40 in Nash bargaining, the Hagedorn and Manovskii model generates a steady-state, which is radically different than those of the Shimer and the Pissarides models. Further, in this setting, job-matching probability in the Hagedorn and Manovskii model turns out to ill-behave, since the steady- state of q reaches a value above 1. Further, this specification yields an unemployment rate u as much as 3.5 times that of the Shimer model, and a vacancy posting rate v of only one-fifth of the Shimer model. Accordingly, labor market tightness in Hagedorn and Manovskii model with low high bargaining power of workers turns out to be significantly lower than that of Shimer or Pissarides models. Therefore, it is safe to conclude that for the Hagedorn and Manovskii model to perform comparably to the Shimer and Pissarides models, it is imperative to set the worker's bargaining power significantly lower, as high values of nonmarket activity already empower workers and thereby reduce opportunity cost of working.

Table 2

Steady-States of the Three Models

	u	z	c	v	p	q	χ	w	R	θ	n	y
Shimer	0.0957	0.0000	0.8699	0.2765	0.9450	0.3271	0.3626	0.9065	1.0101	2.8894	0.9043	0.9043
Hagedorn and Manovskii	0.0950	0.0000	0.9172	0.2781	0.9525	0.3254	0.8607	0.9060	1.0101	2.9274	0.9050	0.9050
Pissarides	0.0923	0.0000	0.8709	0.2850	0.9837	0.3184	0.3731	0.9328	1.0101	3.0891	0.9077	0.9077

u denotes unemployment rate, z denotes total factor productivity (in exponential form), c denotes consumption, v denotes vacancy postings, p denotes the rate at which jobs are filled, q denotes the rate at which vacancies are filled, χ denotes unemployment benefit/ home production determined as a ratio of the wage rate, w denotes the wage rate, R denotes the gross rate of return, θ denotes labor market tightness, n denotes labor and y denotes output.

After computing the deterministic steady-states, I next turn to deriving decision rules and law of motions in the neighborhood of the respective steady-states of the three models in response to labor productivity shocks. For this goal, I utilize the Schmitt-Grohé and Uribe first-order linear local approximation algorithm. (17) In doing so, as briefly discussed, I follow two different approaches: i) I formulate nonmarket activity χ as a *parameter*, the unique value of which is as listed in Table 2, or ii) I *endogenize* the value of nonmarket activity so that the ratio of nonmarket activity to wage rate remains constant not only at the steady-state, but also throughout business-cycle fluctuations resulting from productivity shocks. I coin the former specification as *exogenous* χ and the latter as *endogenous* χ. (18) For each of the three competing models, I compare and contrast the implications of the endogeneity of nonmarket activity, χ. (19)

Since the main focus of interest of labor-search models is addressing discrepancies in second moments, I next turn to simulating model economies so as to explore on second moments. For this purpose, I simulate 100 model economies with different productivity innovations, while keeping innovations the same across models. After generating the time-series of economic variables of interest around their respective steady-states for 100 quarters, I next take the natural logarithm of the computed series, and apply the Hodrick and Prescott filter to generate the trend and de-trended series. (20) I display the descriptive statistics of the resultant series in Table 3, Table 4 and Table 5.

(17) I append the details of the local approximation algorithm to the Appendix.

(18) For robustness check on decision rules and law of motions, I shut down exogenous innovations to labor productivity for 99 periods: $t = 2, \ldots, 100$ and feed the three models with an initial productivity innovation of positive 2 standard deviations in their respective steady-states at time $t = 1$. For brevity, I display the convergence patterns of only one state variable, u_t, and one control variable, v_t in Figure A.1, Figure A.2 and Figure A.3. In summary, all variables illustrate a sharp and clear convergence pattern to their long run means, with the endogeneity of χ inducing noticable differences.

(19) While I set χ as exogenously for comparability with the earlier literature, my rationale for endogenizing χ is due to reporting on the considerable implications of incorporating co-movement of nonmarket activity and wage rate throughout business-cycles. As I discuss in detail, this distinction has first-order implications, which is not studied in the earlier literature.

(20) While using Hodrick and Prescott filter, I use a smoothing parameter λ equal to 1600, as it is standard in the macroeconomics literature. Note that Shimer (2005) sets the smoothing parameter equal to 10^5, thereby condensing the responses to innovations more than the common practice in the literature.

Table 3
Descriptive Statistics of the Shimer Model

Endogenous χ

	u	e^z	c	v	p	q	w	R	θ	y
Std. Deviation	0.0076	0.0088	0.0080	0.0054	0.0056	0.0042	0.0083	0.0005	0.0458	0.0082
Rel. Std. (e^z)	0.8598	1.0000	0.9091	0.6143	0.6347	0.4774	0.9421	0.0587	5.2053	0.9330
Rel. Std. (y)	0.9215	1.0718	0.9743	0.6584	0.6803	0.5117	1.0097	0.0630	5.5789	1.0000
Corr.(e^z)	0.1332	1.0000	0.9675	0.1789	0.9889	-0.1682	0.9838	-0.4070	0.7599	0.9833
Corr.(y)	-0.0267	0.9833	0.9970	0.3230	0.9941	-0.3239	0.9998	-0.4282	0.6456	1.0000
Autocorr.	0.9738	0.6689	0.6966	0.8572	0.7305	0.9373	0.6923	0.4979	0.8775	0.6890

Exogenous χ

	u	e^z	c	v	p	q	w	R	θ	y
Std. Deviation	0.0119	0.0088	0.0076	0.0085	0.0088	0.0066	0.0079	0.0007	0.0718	0.0084
Rel. Std. (e^z)	1.3473	1.0000	0.8619	0.9624	0.9948	0.7480	0.8985	0.0809	8.1659	0.9568
Rel. Std. (y)	1.4081	1.0451	0.9008	1.0059	1.0397	0.7818	0.9391	0.0846	8.5345	1.0000
Corr.(e^z)	0.1332	1.0000	0.9648	0.1789	0.9886	-0.1682	0.9831	-0.0783	0.7599	0.9818
Corr.(y)	-0.0276	0.9818	0.9968	0.3185	0.9956	-0.3241	0.9998	-0.1326	0.6468	1.0000
Autocorr.	0.9738	0.6689	0.7164	0.8569	0.7304	0.9372	0.6999	0.2239	0.8774	0.7038

Table 4
Descriptive Statistics of the Hagedorn and Manovskii Model

Endogenous χ

	u	e^z	c	v	p	q	w	R	θ	y
Std. Deviation	0.0076	0.0088	0.0082	0.0054	0.0057	0.0042	0.0084	0.0004	0.0469	0.0082
Rel. Std. (e^z)	0.8618	1.0000	0.9356	0.6167	0.6512	0.4796	0.9500	0.0496	5.3309	0.9345
Rel. Std. (y)	0.9222	1.0701	1.0011	0.6599	0.6968	0.5132	1.0166	0.0531	5.7044	1.0000
Corr.(e^z)	0.1328	1.0000	0.9879	0.1858	0.9936	-0.1718	0.9844	-0.6725	0.7653	0.9836
Corr.(y)	-0.0259	0.9836	0.9995	0.3266	0.9950	-0.3256	0.9998	-0.6842	0.6515	1.0000
Autocorrelation	0.9736	0.6689	0.6838	0.8388	0.7107	0.9338	0.6825	0.6956	0.8675	0.6894

Exogenous χ

	u	e^z	c	v	p	q	w	R	θ	y
Std. Deviation	0.0676	0.0088	0.0051	0.0482	0.0509	0.0374	0.0038	0.0028	0.4388	0.0115
Rel. Std. (e^z)	7.6821	1.0000	0.5813	5.4809	5.7832	4.2517	0.4291	0.3145	49.8804	1.3032
Rel. Std. (y)	5.8949	0.7673	0.4460	4.2058	4.4377	3.2626	0.3293	0.2414	38.2757	1.0000
Corr.(e^z)	0.1338	1.0000	0.8640	0.1835	0.9757	-0.1698	0.9832	0.3829	0.7585	0.9395
Corr.(y)	-0.0332	0.9395	0.9774	0.2635	0.9656	-0.3120	0.9720	0.1007	0.6372	1.0000
Autocorrelation	0.9736	0.6689	0.8139	0.8391	0.7170	0.9337	0.7013	-0.0459	0.8668	0.8008

Table 5
Descriptive Statistics of the Pissarides Model

Endogenous χ

	u	e^z	c	v	p	q	w	R	θ	y
Std. Deviation	0.0053	0.0088	0.0079	0.0036	0.0038	0.0029	0.0082	0.0004	0.0336	0.0081
Rel. Std. (e^z)	0.6003	1.0000	0.8945	0.4113	0.4311	0.3322	0.9288	0.0438	3.8189	0.9202
Rel. Std. (y)	0.6524	1.0867	0.9721	0.4470	0.4685	0.3610	1.0093	0.0476	4.1501	1.0000
Corr.(e^z)	0.1375	1.0000	0.9692	0.1659	0.9626	-0.1260	0.9913	-0.7888	0.7116	0.9851
Corr.(y)	-0.0147	0.9851	0.9967	0.3113	0.9579	-0.2775	0.9984	-0.7835	0.5997	1.0000
Autocorrelation	0.9748	0.6689	0.6824	0.9021	0.8047	0.9520	0.6922	0.8513	0.9152	0.6779

Exogenous χ

	u	e^z	c	v	p	q	w	R	θ	y
Std. Deviation	0.0083	0.0088	0.0074	0.0057	0.0060	0.0046	0.0078	0.0004	0.0530	0.0082
Rel. Std. (e^z)	0.9480	1.0000	0.8390	0.6491	0.6793	0.5245	0.8846	0.0452	6.0290	0.9345
Rel. Std. (y)	1.0145	1.0701	0.8979	0.6947	0.7269	0.5613	0.9466	0.0483	6.4519	1.0000
Corr.(e^z)	0.1376	1.0000	0.9680	0.1649	0.9611	-0.1249	0.9903	-0.5034	0.7100	0.9844
Corr.(y)	-0.0151	0.9844	0.9966	0.3088	0.9608	-0.2775	0.9984	-0.5205	0.6007	1.0000
Autocorrelation	0.9748	0.6689	0.6944	0.9034	0.8065	0.9523	0.7017	0.7074	0.9161	0.6878

Shimer (2005), in his canonical paper, argues that *"In the United States, the standard deviation of the vacancy-unemployment ratio is almost 20 times as large as the standard deviation of average labor productivity, while the search model predicts that the two variables should have nearly the same volatility"*. The relative volatility of θ to e^z generated in my Shimer specifications is neither nearly equal to unity, nor is it close to 20.

When nonmarket activity, χ_t, is specified endogenously, the standard deviation of labor market tightness θ equals 521% of the standard deviation of labor productivity, e^z. The same ratio increases up to 817% when modelling χ exogenously. A critical result generated by the Shimer specification, as well as the other two specifications, is that whenever χ is endogenized, i.e. nonmarket activity is time and state-independent, the relative volatility of labor market tightness decreases, and formulating nonmarket exogenously, as done in the earlier literature, is therefore a critical element in amplifying the volatility of labor market tightness. [21]

In order to quantify the impact of Hodrick and Prescott smoothing parameter on these results, I set λ equal to 10^5 as in Shimer (2005), and report my findings in Table A.4 and Table A.5: as expected, increasing the smoothing parameter generates a lower relative labor market tightness volatility, yield- ing values closer to those by Shimer (2005). In this setting, increasing the smoothing parameter from the conventional $\lambda = 1600$ to $\lambda = 10^5$ reduces the volatility of labor market tightness by as much as one fourth: from

(21) Note that the negative correlation between unemployment and vacancies, as displayed in Table A.1, Table A.2 and Table A.3 verify that the all three models generate downward-sloping Beveridge curves, with comparable figures to those reported by Shimer (2005).

521% to 398% for the endogenous χ case, and from 817% to 624% for the exogenous χ case. Overall, it is possible to argue that the choice of Hodrick and Prescott smoothing parameter has sizable implications in favor of the Shimer puzzle. [22]

Table 4 verifies the claim by Hagedorn and Manovskii (2008) that the Hagedorn and Manovskii model with exogenous nonmarket activity does an excellent job in amplifying the relative standard deviation of labor market tightness with its value of 4988%, which is over six times that of the Shimer model. [23] However, this result does not hold true when nonmarket activity χ is endogenized: in the case of endogenous χ, the relative volatility prediction of the Hagedorn and Manovskii on labor market tightness is just 533%, which is only moderately over that of the Shimer specification with 521%. These findings suggest that the success of the Hagedorn and Manovskii (2008) in amplifying the relative volatility of interest, and thereby addressing the Shimer puzzle depends considerably on the formulation of nonmarket activity: if the value of nonmarket activity (i.e. unemployment bene- fits/home production) is allowed to co-move with the wage rate in response to technology shocks, the suggested solution by Hagedorn and Manovskii (2008) to the Shimer puzzle fails to deliver. In other words, Hagedorn and Manovskii (2008)'s strength lies decisively on the assumption of constant non- market activity, thus further research addressing whether nonmarket activity is indeed empirically irresponsive to productivity innovations is key.[24]

I report my findings on the second moments of variables of interest in the Pissarides model in Table 5. Evidently, the descriptive statistics by the Pissarides specification compare rather poorly to the Shimer and the Hagedorn and Manovskii specifications: the volatility ratio of labor market tightness productivity shocks is lower than that of the Shimer and Hagedorn and Manovskii specifications for both the endogenous and the exogenous nonmarket activity settings. When the concavity parameter ι is reduced to 0.30 so that the vacancy posting function decreases more to scales, relative volatility ratio improves, yet still falls short of delivering relative volatility figures close to those of the Shimer specifications.[25] Therefore, in the absence of ad-hoc wage stickiness, the Pissarides model with decreasing average cost of posting vacancies, as formulated in Equation (29) does not offer promising outcomes, and further research extending Pissarides (2009) could focus on the re- formulation the posting cost function, while simultaneously incorporating other vital elements, such as wage stickiness. [26]

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(22) For a more elaborate discussion on this issue, see Shimer (2010).

(23) In fact, it overamplifies by twofolds, as the relevant figure in the data is approximately 2000%.

(24) Note throughout this exercise, I assume that the worker's bargaining power is 5%, which is almost an order of magnitude lower than that of the competing two models with ν = 0.40. In the Hagedorn and Manovskii model, when the worker's bargaining power is set to the same ν = 0.40, the model starts to display ill-behaving results, with the probability of filling a vacancy exceeding unity, and unemployment rate reaching over one-third of the population, as displayed in Table A.6. Therefore, for the Hagedorn and Manovskii model to generate a decent performance, high nonmarket activity is to be coupled with unreasonably low bargaining power of workers, as well.

(25) In this experiment, I re-calibrate the coefficient before cost of vacancies, i.e. 0.6019, to 0.3600, so that $\bar{\psi} = 0.3600\,\bar{v}^{0.3} = 0.25$ holds thereby ensuring comparability of the two specifications. See Table A.9, Table A.10 and Table A.11 for the details of the findings by this re-parametrization.

(26) For further concerns on Pissarides (2009), see Silva and Toledo (2013).

Conclusions

In this paper, I explore on the business-cycle performances of three prominent labor-search models: Shimer (2005), Hagedorn and Manovskii (2008) and Pissarides (2009). I first provide a standard framework for contrasting the three models, and quantify the sensitivity of Shimer (2005)'s relative labor market tightness volatility to business cycle smoothing parametrization. Second, I revisit Hagedorn and Manovskii (2008)'s proposed solution to the Shimer puzzle, and report that the strength of the model by Hagedorn and Manovskii (2008) depends considerably on whether value of nonmarket activity responds to business-cycle fluctuations or not. Third, I report that when specified in the form of a decreasing-returns-to-scale functional form, the incorporation of decreasing average vacancy posting costs by Pissarides (2009) cannot generate decent business-cycle volatilities in the absence of further wage stickiness. All my findings reveal the way nonmarket activity is modelled has key implications for business-cycle performances of labor-search models. Specifically, formulating the value of nonmarket activity as a constant, or allowing it to respond to business-cycle fluctuations has first-order consequences, especially in settings where bargaining power of workers is low and the ratio of nonmarket activity to wage rate is high. Therefore, these findings point to the need for exploring further on the degree of co-movement between wage rate and nonmarket activity in order to improve on labor-search modelling attempts, which I leave to future research.

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Appendix

A Appendix Tables and Figures

Tables

Table A.1

Correlation Matrix in the Shimer Model

Endogenous χ

	u	v	θ	p	q
u	1.0000	-0.8981	0.7097	0.0453	0.9409
v	-0.8981	1.0000	-0.4631	0.2387	-0.9719
θ	0.7097	-0.4631	1.0000	0.7045	0.4665
p	0.0453	0.2387	0.7045	1.0000	-0.2520
q	0.9409	-0.9719	0.4665	-0.2520	1.0000

Exogenous χ

	u	v	θ	p	q
u	1.0000	-0.8980	0.7096	0.0448	0.9409
v	-0.8980	1.0000	-0.4627	0.2394	-0.9718
θ	0.7096	-0.4627	1.0000	0.7041	0.4663
p	0.0448	0.2394	0.7041	1.0000	-0.2524
q	0.9409	-0.9718	0.4663	-0.2524	1.0000

Table A.2

Correlation Matrix in the Hagedorn and Manovskii Model

Endogenous χ

	u	v	θ	p	q
u	1.0000	-0.8899	0.7071	0.0579	0.9396
v	-0.8899	1.0000	-0.4467	0.2395	-0.9694
θ	0.7071	-0.4467	1.0000	0.7166	0.4601
p	0.0579	0.2395	0.7166	1.0000	-0.2431
q	0.9396	-0.9694	0.4601	-0.2431	1.0000

Exogenous χ

	u	v	θ	p	q
u	1.0000	-0.8867	0.7014	0.0344	0.9395
v	-0.8867	1.0000	-0.4378	0.2659	-0.9646
θ	0.7014	-0.4378	1.0000	0.6913	0.4568
p	0.0344	0.2659	0.6913	1.0000	-0.2612
q	0.9395	-0.9646	0.4568	-0.2612	1.0000

Table A.3

Correlation Matrix in the Pissarides Model

Endogenous χ

	u	v	θ	p	q
u	1.0000	-0.9183	0.7408	0.1212	0.9528
v	-0.9183	1.0000	-0.5267	0.1385	-0.9782
θ	0.7408	-0.5267	1.0000	0.7286	0.5337
p	0.1212	0.1385	0.7286	1.0000	-0.1437
q	0.9528	-0.9782	0.5337	-0.1437	1.0000

Exogenous χ

	u	v	θ	p	q
u	1.0000	-0.9190	0.7415	0.1213	0.9530
v	-0.9190	1.0000	-0.5286	0.1374	-0.9784
θ	0.7415	-0.5286	1.0000	0.7278	0.5352
p	0.1213	0.1374	0.7278	1.0000	-0.1429
q	0.9530	-0.9784	0.5352	-0.1429	1.0000

Table A.4

Descriptive Statistics of the Shimer Model with $\lambda = 10^5$

Endogenous χ

	u	e^z	c	v	p	q	w	R	θ	y
Std. Deviation	0.0050	0.0131	0.0121	0.0041	0.0086	0.0032	0.0126	0.0007	0.0522	0.0124
Rel. Std. (e^z)	0.3808	1.0000	0.9209	0.3138	0.6548	0.2477	0.9582	0.0546	3.9790	0.9481
Rel. Std. (y)	0.4017	1.0548	0.9713	0.3310	0.6907	0.2612	1.0107	0.0576	4.1969	1.0000
Corr.(e^z)	0.0136	1.0000	0.9925	0.4644	0.9955	-0.5258	0.9961	-0.6716	0.9134	0.9960
Corr.(y)	-0.0593	0.9960	0.9993	0.5164	0.9979	-0.5875	0.9999	-0.6911	0.8817	1.0000
Autocorr.	0.9596	0.8264	0.8426	0.7710	0.8629	0.9087	0.8417	0.7092	0.8985	0.8398

Exogenous χ

	u	e^z	c	v	p	q	w	R	θ	y
Std. Deviation	0.0078	0.0131	0.0115	0.0064	0.0134	0.0051	0.0120	0.0009	0.0818	0.0128
Rel. Std. (e^z)	0.5966	1.0000	0.8795	0.4918	1.0258	0.3880	0.9165	0.0653	6.2411	0.9776
Rel. Std. (y)	0.6103	1.0229	0.8996	0.5031	1.0493	0.3969	0.9375	0.0668	6.3842	1.0000
Corr.(e^z)	0.0137	1.0000	0.9912	0.4642	0.9954	-0.5259	0.9957	-0.3957	0.9134	0.9953
Corr.(y)	-0.0606	0.9953	0.9992	0.5110	0.9987	-0.5877	0.9999	-0.4376	0.8826	1.0000
Autocorr.	0.9595	0.8264	0.8536	0.7708	0.8627	0.9086	0.8460	0.4394	0.8984	0.8480

† $\lambda = 10^5$ refers to the Hodrick and Prescott filter smoothing parameter in Shimer (2005).

Table A.5

Correlation Matrix in the Shimer Model with $\lambda = 10^5$

Endogenous χ

	u	v	θ	p	q
u	1.0000	-0.7712	0.3611	-0.0306	0.8073
v	-0.7712	1.0000	0.1020	0.4738	-0.9491
θ	0.3611	0.1020	1.0000	0.8995	-0.1816
p	-0.0306	0.4738	0.8995	1.0000	-0.5596
q	0.8073	-0.9491	-0.1816	-0.5596	1.0000

Exogenous χ

	u	v	θ	p	q
u	1.0000	-0.7711	0.3609	-0.0310	0.8072
v	-0.7711	1.0000	0.1020	0.4742	-0.9490
θ	0.3609	0.1020	1.0000	0.8992	-0.1818
p	-0.0310	0.4742	0.8992	1.0000	-0.5601
q	0.8072	-0.9490	-0.1818	-0.5601	1.0000

† $\lambda = 10^5$ refers to the Hodrick and Prescott filter smoothing parameter in Shimer (2005).

Table A.6
Steady-State of the Hagedorn and Manovskii Model with *ν* = 0.40

u	*z*	*c*	*v*	*p*	*q*	*χ*	*w*	*R*	*θ*	*n*	*y*
0.3664	0.0000	0.9556	0.0624	0.1729	1.0148	0.9214	0.9699	1.0101	0.1704	0.6336	0.6336

† ν = 0.40 refers to the worker's bargaining power in Nash bargaining.

Table A.7
Descriptive Statistics of the Hagedorn and Manovskii Model with *ν* = 0.40

Endogenous χ

	u	e^z	*c*	*v*	*p*	*q*	*w*	*R*	*θ*	*y*
Std. Deviation	0.0043	0.0088	0.0084	0.0120	0.0095	0.0052	0.0086	0.0004	0.0158	0.0081
Rel. Std. (e^z)	0.4884	1.0000	0.9525	1.3589	1.0751	0.5898	0.9810	0.0457	1.7975	0.9156
Rel. Std. (y)	0.5335	1.0922	1.0403	1.4842	1.1742	0.6442	1.0715	0.0499	1.9632	1.0000
Corr.(e^z)	0.0227	1.0000	0.9967	-0.1331	-0.0266	-0.9995	0.9983	-0.9961	-0.0294	0.6491
Corr.(y)	-0.6957	0.6491	0.7035	0.6041	0.7017	-0.6319	0.6854	-0.6340	0.6997	1.0000
Autocorr.	0.9654	0.6689	0.6677	0.9724	0.9649	0.6756	0.6646	0.7025	0.9652	0.8113

Exogenous χ

	u	e^z	*c*	*v*	*p*	*q*	*w*	*R*	*θ*	*y*
Std. Deviation	0.0526	0.0088	0.0041	0.1551	0.1197	0.0617	0.0060	0.0005	0.2000	0.0249
Rel. Std. (e^z)	5.9768	1.0000	0.4658	17.6326	13.6115	7.0149	0.6863	0.0594	22.7403	2.8356
Rel. Std. (y)	2.1078	0.3527	0.1643	6.2183	4.8003	2.4739	0.2420	0.0210	8.0197	1.0000
Corr.(e^z)	0.0243	1.0000	0.9418	-0.1349	-0.0299	-0.9826	0.9985	0.6666	-0.0292	0.3523
Corr.(y)	-0.8482	0.3523	0.5232	0.6534	0.7406	-0.3525	0.3929	-0.0096	0.7416	1.0000
Autocorr.	0.9656	0.6689	0.7898	0.9727	0.9652	0.6812	0.6664	0.5444	0.9651	0.9352

† ν = 0.40 refers to the worker's bargaining power in Nash bargaining.

Table A.8
Correlation Matrix in the Hagedorn and Manovskii Model with ν = 0.40

Endogenous χ

	u	*v*	θ	*p*	*q*
u	1.0000	-0.8496	-0.9183	-0.9183	-0.0423
v	-0.8496	1.0000	0.9790	0.9787	0.1556
θ	-0.9183	0.9790	1.0000	1.0000	0.0526
p	-0.9183	0.9787	1.0000	1.0000	0.0498
q	-0.0423	0.1556	0.0526	0.0498	1.0000

Exogenous χ

	u	*v*	*θ*	*p*	*q*
u	1.0000	-0.8400	-0.9077	-0.9086	-0.0049
v	-0.8400	1.0000	0.9783	0.9782	0.1279
θ	-0.9077	0.9783	1.0000	0.9991	0.0227
p	-0.9086	0.9782	0.9991	1.0000	0.0240
q	-0.0049	0.1279	0.0227	0.0240	1.0000

† ν = 0.40 refers to the worker's bargaining power in Nash bargaining.

Table A.9

Steady-State of the Pissarides Model with ι = 0.30

u	z	c	v	p	q	χ	w	R	θ	n	y
0.0880	0.0000	0.8720	0.2967	1.0370	0.3074	0.3881	0.9702	1.0101	3.3729	0.9120	0.9120

† ι = 0.30 refers to the exponent of cost of posting vacancies.

Table A.10

Correlation Matrix in the Pissarides Model with ι = 0.30

Endogenous χ

	u	v	θ	p	q
u	1.0000	-0.8990	0.7403	0.1993	0.9535
v	-0.8990	1.0000	-0.4909	0.0976	-0.9709
θ	0.7403	-0.4909	1.0000	0.7837	0.5345
p	0.1993	0.0976	0.7837	1.0000	-0.0632
q	0.9535	-0.9709	0.5345	-0.0632	1.0000

Exogenous χ

	u	v	θ	p	q
u	1.0000	-0.8998	0.7409	0.1993	0.9538
v	-0.8998	1.0000	-0.4928	0.0964	-0.9712
θ	0.7409	-0.4928	1.0000	0.7830	0.5358
p	0.1993	0.0964	0.7830	1.0000	-0.0624
q	0.9538	-0.9712	0.5358	-0.0624	1.0000

† ι = 0.30 refers to the exponent of cost of posting vacancies.

Table A.11

Descriptive Statistics of the Pissarides Model with ι = 0.3

Endogenous χ

	u	e^z	c	v	p	q	w	R	θ	y
Std. Deviation	0.0059	0.0088	0.0080	0.0040	0.0047	0.0033	0.0084	0.0004	0.0424	0.0082
Rel. Std. (e^z)	0.6752	1.0000	0.9044	0.4538	0.5355	0.3763	0.9587	0.0492	4.8151	0.9285
Rel. Std. (y)	0.7273	1.0770	0.9740	0.4888	0.5767	0.4053	1.0326	0.0530	5.1859	1.0000
Corr.(e^z)	0.1384	1.0000	0.9695	0.1978	0.9781	-0.1267	0.9972	-0.6367	0.7252	0.9865
Corr.(y)	-0.0064	0.9865	0.9962	0.3317	0.9599	-0.2702	0.9937	-0.6415	0.6190	1.0000
Autocorrelation	0.9744	0.6689	0.6872	0.8671	0.7825	0.9489	0.6964	0.7180	0.9029	0.6806

Exogenous χ

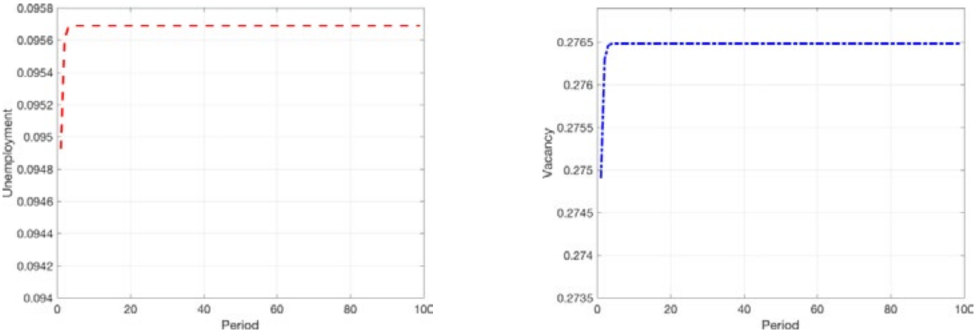
	u	e^z	c	v	p	q	w	R	θ	y
Std. Deviation	0.0097	0.0088	0.0075	0.0065	0.0077	0.0054	0.0082	0.0005	0.0690	0.0083
Rel. Std. (e^z)	1.1008	1.0000	0.8520	0.7391	0.8710	0.6133	0.9362	0.0597	7.8491	0.9466
Rel. Std. (y)	1.1629	1.0565	0.9001	0.7809	0.9202	0.6480	0.9890	0.0631	8.2922	1.0000
Corr.(e^z)	0.1385	1.0000	0.9678	0.1970	0.9770	-0.1258	0.9962	-0.2797	0.7238	0.9856
Corr.(y)	-0.0067	0.9856	0.9960	0.3281	0.9622	-0.2699	0.9937	-0.3123	0.6199	1.0000
Autocorrelation	0.9744	0.6689	0.7032	0.8690	0.7847	0.9493	0.7063	0.4291	0.9039	0.6927

† ι = 0.30 refers to the exponent of cost of posting vacancies.

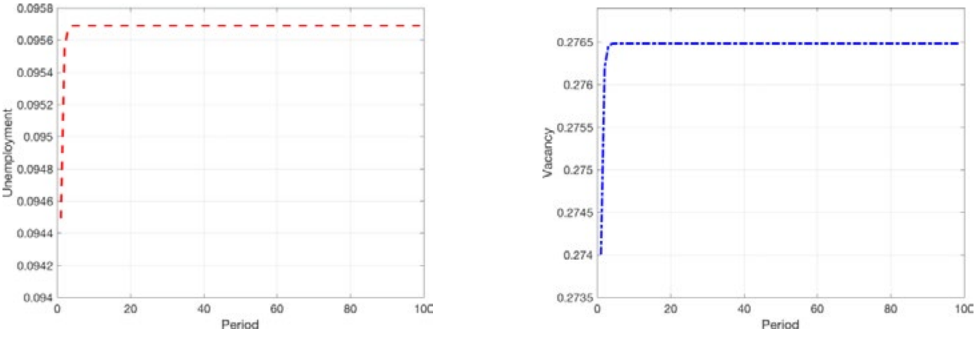
Figures

Figure A.1
Convergence in the Shimer Model

Endogenous χ



Exogenous χ

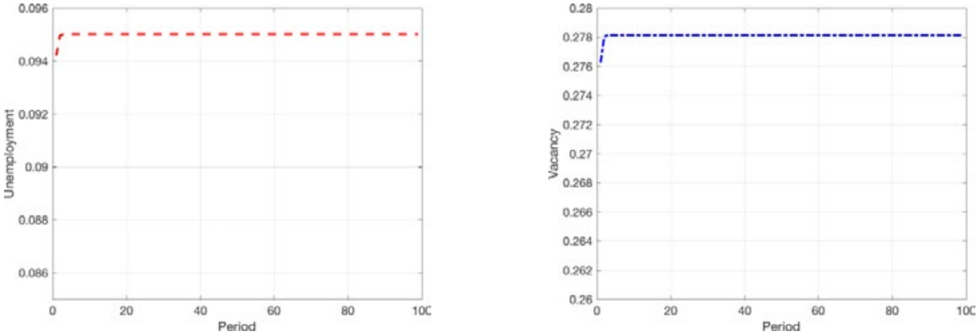


† The red line refers to the response of unemployment (u), and the blue line refers to the response of vacancy (v) to a 2-standard-deviation positive labor productivity shock (ε^z) at the steady-state.

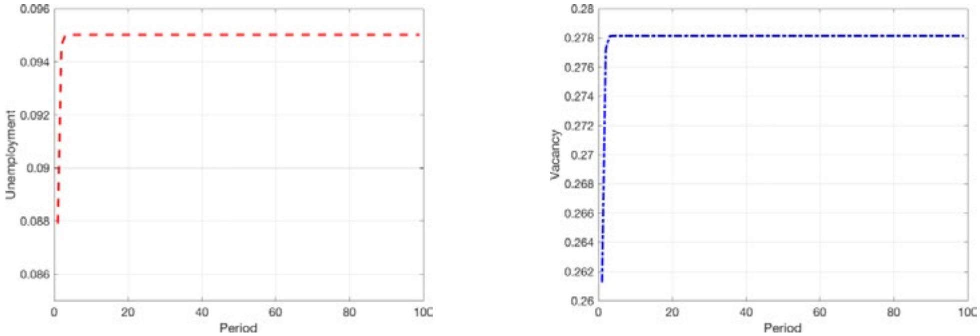
Figure A.2

Convergence in the Hagedorn and Manovskii Model

Endogenous χ



Exogenous χ

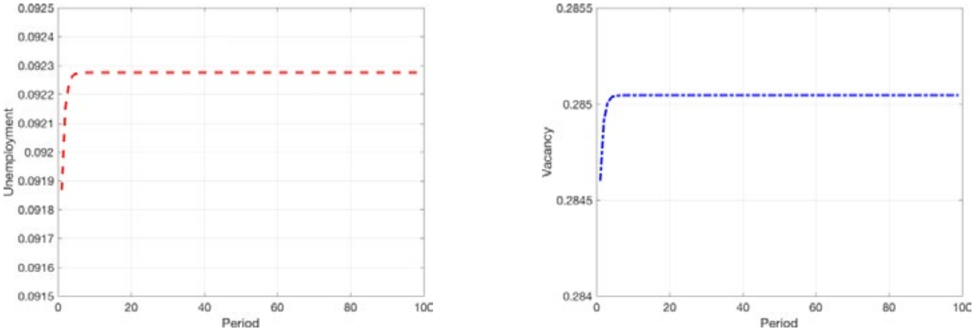


† The red line refers to the response of unemployment (u), and the blue line refers to the response of vacancy (v) to a 2-standard-deviation positive labor productivity shock (ε^z) at the steady-state.

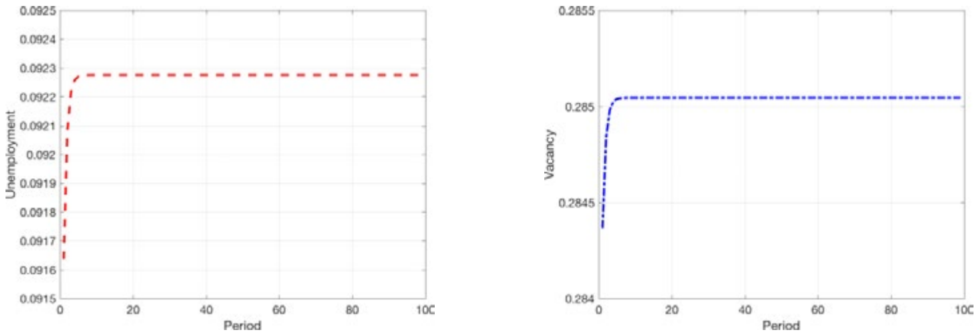
Figure A.3

Convergence in the Pissarides Model

Endogenous χ



Exogenous χ



† The red line refers to the response of unemployment (u), and the blue line refers to the response of vacancy (v) to a 2-standard-deviation positive labor productivity shock (ε^z) at the steady-state.

B Appendix: Solution Algorithm

In this appendix section, I describe the details of the linear approximation algorithm around steady-state. First, I compute the deterministic steady states of the three models by solving the simultaneous system of model equations via the fsolve command in MATLAB, as it is standard in the literature. Next, I turn to exploring the business cycle properties variables of interest via Schmitt- Grohé and Uribe (2004) first-order linear approximation algorithm around steady-states. Let ζ_t be the stacked vector of co-state variables, i.e. $[u_t; z_t]$ and κ_t be the stacked vector of state variables, i.e. $[c_t; v_t; p_t; q_t; \chi_t; w_t; R_t; \Xi_t]$ if χ is endogenized and $[c_t; v_t; p_t; q_t; w_t; R_t; \Xi_t]$ if χ is kept as a parameter. Then, the first-order linear approximations of the decision rules imply:

$$\zeta_t = \varrho(\kappa_t, \sigma) \approx \varrho(\bar{\kappa}, 0) + \varrho_\kappa(\kappa_t - \kappa) + \varrho_\sigma(\kappa, 0)\sigma$$
$$\kappa_{t+1} = \hbar(\kappa_t, \sigma) + \eta\sigma\varepsilon_{t+1} \approx \hbar(\kappa, 0) + \hbar_\kappa(\kappa_t - \kappa) + \hbar_\sigma(\kappa, 0)\sigma + \eta\sigma\varepsilon_{t+1}$$

where σ refers to perturbation from the steady-state, hence $\zeta = \varrho(\kappa, 0), \kappa = \hbar(\kappa, 0)$, and $\hbar_\sigma = 0$ and $\varrho_{\kappa\sigma} = 0$. Equation (30) describes how the control variables rely on state variables, and Equation (31) illustrates how the state variables evolve. Next, I rewrite the equilibrium conditions in a compact way so that expected value of each function as of time t equals 0, and we stack these equations in a vector of functions Γ_t. Then, by chain rule, simple algebra yields:

$$\Gamma_{\zeta_{t+1}}\varrho_\kappa\hbar_\kappa + \Gamma_{\zeta_t}\varrho_\kappa + \Gamma_{\kappa_{t+1}}\hbar_\kappa + \Gamma_{\kappa_t} = 0$$

where Γ_ι refers to the Jacobian of $\Gamma(.)$ with respect to ι. The analytical forms of the Jacobians could be derived with tedious algebra since functional forms are already known, and having solved the deterministic steady-state, evaluating the Jacobians at steady-state values for equation is feasible, which are used to derive ϱ_κ and $\hbar_\kappa$ matrices, i.e. decision rules.