

Liquidity Constraints, Interest Rates and Optimal Credit Subsidies*

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Abstract

There is an ongoing debate about whether credit subsidies improve welfare or not. We build a model of monopolistic competition with liquidity constrained firms to analyze the optimal credit subsidy policy. The contribution of this paper is to determine the role of interest rates on optimal credit subsidies. This paper shows that i) there exists an optimal credit subsidy level which always increases welfare; ii) however, some subsidy levels can decrease welfare compared to no policy case, and iii) the market interest rate plays a crucial role in determining the optimal level for credit subsidy.

Keywords: liquidity constraints, credit subsidies, government policy, welfare, interest rate.

JEL Classification: E60, G28, H32, O11.

Likidite Kısıtları, Faiz Oranları ve Optimal Kredi Sübvansiyonları

Özet

Kredi sübvansiyonlarının refahı artırıp artırmadığına ilişkin devam eden bir tartışma vardır. Bu çalışmada optimal kredi sübvansiyon politikasını analiz etmek için firmaların likidite kısıtı altında olduğu bir monopolistik rekabet modeli oluşturulmuştur. Bu makalenin katkısı, faiz oranlarının optimal kredi sübvansiyonları üzerindeki rolünü analiz etmektir. Bu makalenin üç temel bulgusu şunlardır: i) Her zaman refahı artıran bir optimal kredi sübvansiyonu vardır; ii) fakat, bazı sübvansiyon düzeyleri hiçbir politikanın uygulanmadığı duruma göre refahı azaltabilir, iii) piyasa faiz oranı, optimal kredi sübvansiyon düzeyini belirlemede önemli rol oynamaktadır.

Anahtar kelimeler: likidite kısıtları, kredi sübvansiyonları, hükümet politikası, refah, faiz oranı.

JEL Sınıflaması: E60, G28, H32, O11.

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The efficiency of government interventions has been studied theoretically by Stiglitz and Weiss (1981), De Meza and Webb (1988), Gale (1991), Innes (1991) among others. This paper analyzes the role of government in improving welfare on economies with liquidity constrained firms. We study optimal government policy in terms of credit subsidy in a Dixit-Stiglitz (1977) model of monopolistic competition with liquidity constraints. There is an ongoing debate about whether credit subsidies^[1] improve welfare or not. Some studies show that policy interventions such as credit subsidies have negative effects on welfare (Li 2002, and Antunes, Cavalcanti, Villamil 2015 among others). On the other hand, some studies find that credit subsidies are beneficial for the economy (Smith, Stutzer 1989 and Itskhoki, Moll 2015 among others). Costrell (1990) using the Dixit-Stiglitz (1977) model of monopolistic competition has already shown that subsidy is welfare-improving. However, the main difference of this paper is to focus on the effects of interest rates on optimal credit subsidies. Hence, this paper as a contribution to the existing literature documents that: 1) there exists an optimal credit subsidy level which always increases welfare, 2) some subsidy levels can decrease welfare compared to no policy case, and 3) market interest rates are crucial in determining the optimal credit subsidy level. The first and second results together indicate that the credit subsidy policy can either increase or decrease welfare depending on the level of subsidy.

The remainder of the paper is organized as follows: The next section presents the model. In section 3, we discuss welfare analysis. Section 4 concludes.

The Model

We present a version of the Dixit-Stiglitz (1977) model of monopolistic competition with two modifications. First, in the economy capital markets are assumed to be imperfect. Second, a government levies taxes on labor income to provide credit subsidies for firms.

Preferences

There are L consumers in the economy, each supplying one unit of labor inelastically. Preferences for each identical consumer are defined over a continuum of differentiated varieties indexed by $z \in \Omega$, and a homogeneous good indexed by 0 which is chosen as numeraire. All consumers share the same quasi-linear utility function:^[2]

$$U(q_0, C) = q_0 + \alpha \log(C), \quad (1)$$

^[1] When a government loans money to a company at a lower rate of interest than a commercial bank would offer, the company saves money. We name the difference between the market interest rate and the interest rate that a firm faces as a credit subsidy.

^[2] We need the quasi-linear utility function to achieve our goals. Without the numeraire good, there is no role for subsidy (for example, standard CES utility without homogeneous good) to improve the welfare.

This economy maximizes the utility function, subject to budget constraint:^[3]

$$q_0 + PC = w(1 - \tau)L, \quad (2)$$

where q_0 represents the consumption level of the numeraire good that is produced under perfect competition. The price of the numeraire good is normalized to 1. The unit input requirement for the numeraire good is one, which then implies that the nominal wage, w , is equal to one.

$C \equiv \left(\int_{z \in \Omega} q(z) \frac{\sigma-1}{\sigma} dz \right)^{\frac{\sigma}{\sigma-1}}$ denotes the composite good, where P is the price of the composite good, $q(z)$ is the quantity demanded of each variety z , and $\sigma > 1$ is the elasticity of substitution across goods. α is the positive demand parameter measuring the preference for the differentiated varieties with respect to the numeraire good. All consumers pay tax on their labor income at a constant rate $\tau \in (0,1)$.

The solution of the utility maximization problem implies the following demand functions for the numeraire good^[4] and each variety z , respectively:

$$q_0 = L - \tau L - \alpha, \quad (3)$$

$$q(z) = \frac{p(z)^{-\sigma}}{P^{1-\sigma}} \alpha, \quad (4)$$

where $p(z)$ is the price of the good z , and the price index, P , is given by

$$P \equiv \left(\int_{z \in \Omega} p(z)^{1-\sigma} dz \right)^{\frac{1}{1-\sigma}}. \quad (5)$$

Welfare can be expressed using the indirect utility function associated with (1)

$$V = (1 - \tau)L + \alpha(\log(\alpha) - 1) - \alpha \log(P). \quad (6)$$

Firms

Each differentiated variety is produced by a different firm where n is the mass of the operating firms which is endogenously determined in the equilibrium. All firms have common productivity φ and share the same production technology: $y = \varphi l$ where y is the output, and l is the amount of labor used by a firm. A sunk cost of entry, f_e , is paid by each firm before production.

Firms face liquidity constraints in financing their production cost. More precisely, firms borrow their production cost in advance at a market interest rate $r \in (0,1)$ which

^[3] In this maximization problem, the (decentralized) economy is populated by a mass L of identical consumers (each has one unit of labor endowment). This environment is identical to an economy with a representative consumer, which supplies L units of labor endowment.

^[4] To have a positive demand for the numeraire good, we assume $(1 - \tau)L > \alpha$.

is exogenously given.^[5]

However, firms receive a credit subsidy, $s \in [0, r]$ on the market interest rate, r . All costs and payments are paid in terms of labor.

Given demand, market interest rate, and credit subsidy, the problem of the firm is given by^[6]

$$\begin{aligned} \max_p \quad & pq - (1 + r - s) \frac{q}{\varphi} \\ \text{s. t.} \quad & q = \frac{p^{-\sigma}}{\rho^{1-\sigma}} \alpha. \end{aligned}$$

We can drop z from our notation since firms are symmetric. The first term in the profit maximization problem, pq , is the firm's total revenue. The second term, $(1 + r - s) \frac{q}{\varphi}$, is the sum of net payments after subsidy. Hence, a firm faces an interest rate level $(r - s)$ effectively where $s \in [0, r]$ is the credit subsidy.

Profit maximization implies that optimal pricing for the firm is

$$p = \frac{\sigma}{\sigma - 1} \frac{(1 + r - s)}{\varphi}. \quad (7)$$

We need a condition in order to have nonnegative prices: $s \in [0, r]$. According to the equation (7), an increase in r raises the price; on the contrary, an increase in either s or φ decreases the price.

Finally, total profit (before paying the sunk cost) of the firm can be written as

$$\Pi = \frac{q}{\varphi} (1 + r - s) \frac{1}{\sigma - 1}. \quad (8)$$

Government

A government collects taxes on labor income to subsidize credit for the firms. A government budget constraint is then given by

$$G \equiv \frac{nqs}{\varphi} = \tau L, \quad (9)$$

^[5] For simplicity, we assume that firms are borrowing from the outside world. Therefore, interest rates are not affected by the firms in the model. Furthermore, our welfare definition does not include "lenders" since they are from the outside world (i.e. foreign lenders do not affect the welfare of the domestic country).

^[6] The firm's problem can be modeled as

$$\begin{aligned} \max_p \quad & pq - (1 - k) \frac{q}{\varphi} - B \\ \text{s. t.} \quad & q = \frac{p^{-\sigma}}{\rho^{1-\sigma}} \alpha; \quad B = (1 + r - s) k \frac{q}{\varphi}, \end{aligned}$$

where $k \in (0, 1)$ is the fraction of the total cost that cannot be financed internally and must be borrowed. All qualitative results remain unchanged with this extension, however with a less tractable version. Analyses can be presented upon request.

where $\frac{nqs}{\varphi}$ is the amount of the total loans subsidized by a government which is the only source of government spending, G . The total tax income of the government is represented by τL .

Tax rate can be computed using the government budget constraint as a function of credit subsidy level, s :

$$\tau = \frac{\alpha s}{L(1+r-s)} \frac{\sigma-1}{\sigma}. \quad (10)$$

According to the equation (10), taxes are strictly increasing in credit subsidies^[7] at an increasing rate. Technically, subsidies reach their maximum level at $\tau = 1 - \frac{\alpha}{L}$. Hence, the maximum credit subsidy^[8] that a government can implement is obtained as

$$s^{max} = \frac{\sigma(1+r)(L-\alpha)}{L\sigma-\alpha}. \quad (11)$$

Equilibrium

In our model, equilibrium is determined by the free entry and the labor market clearing conditions. The final equilibrium condition, stating that the total income of the economy must equal to the total spending in the economy, automatically holds by Walras' Law.

First, free entry requires that firms keep entering the market until their net profits (after paying sunk cost) are equal to zero. Then, the free entry condition implies the output of a firm:

$$q = \frac{f_e \varphi}{(1+r-s)} (\sigma-1). \quad (12)$$

Second, the labor market clearing condition equalizes the total labor demand to labor supply. Hence, labor market clearing condition can be written as^[9]

$$\begin{aligned} n \left(\frac{(1+r-s)q}{\varphi} + f_e \right) + q_0 &= L - \tau L \Leftrightarrow \\ n &= \frac{L - \tau L - q_0}{f_e \sigma} = \frac{\alpha}{f_e \sigma}. \quad (9) \end{aligned} \quad (13)$$

[7] $\frac{\partial \tau}{\partial s} > 0$; $\frac{\partial^2 \tau}{\partial s^2} > 0$.

[8] In order to guarantee that $s^{max} > 0$, we assume that $\alpha < L$.

[9] We assume that the number of firms is sufficiently large enough. Moreover, note that higher σ (higher substitutability across goods) implies a lower number of firms.

Welfare Analysis

In this section, we present the main findings of the paper. In particular, we analyze the welfare effects of government policies.

1. There exists an optimal credit subsidy level which always increases welfare.

In order to investigate whether government policy increases welfare or not, we focus on the following equation

$$\frac{\partial V}{\partial s} = \frac{\alpha(r - s\sigma + 1)}{\sigma(r - s + 1)^2}, \quad (14)$$

where V is the indirect utility function.

By setting $\frac{\partial V}{\partial s} = 0$, optimal credit subsidy can be obtained as

$$s^* = \frac{r + 1}{\sigma} > 0. \quad (15)$$

Recall that s^* cannot exceed r to have non-negative prices. However, according to equation (15), for some combinations of σ and r , the values for s^* can exceed the values of r . In these cases, instead of the first best, namely equation (15), the policy maker should set the second best policy, namely $s^* = r$. When σ is small enough (i.e. the market is highly concentrated), subsidies must be large to improve welfare as seen in Table 1.

Table 1 presents the examples of optimal credit subsidy and tax rate^[10] values under alternative combinations for σ and r . As we can see from Table 1, for some combinations of σ and r , s^* exceeds r . In these cases, optimal s^* should be equal to r . Most of the time, these cases are valid for relatively small interest rates (and for relatively small σ values).

Table 1:
Values of Optimal Credit Subsidies and Corresponding Tax Rates

$\sigma \backslash r$	0.01	0.05	0.1	0.2	0.4	0.6	τ^*
2	0.505	0.525	0.55	0.6	0.7	0.8	0.005
5	0.202	0.21	0.22	0.24	0.28	0.32	0.002
8	0.126	0.131	0.138	0.15	0.175	0.2	0.00125
12	0.084	0.088	0.092	0.1	0.117	0.133	0.00083
20	0.051	0.053	0.055	0.06	0.07	0.08	0.0005

As a conclusion, for relatively small values of interest rates, the optimal policy is to fully subsidize the interest rate, and for relatively large interest rate values (for sufficiently large σ), it is optimal to subsidize the portion of the interest rate. For example, for $\sigma = 8$, if

^[10] Values of optimal tax rates are computed for $\alpha = 100$ and $L = 10000$.

$r = 0.01$, then equation (15) implies that $s^* = 0.126$. However, since $s^* = 0.126$ is greater than $r = 0.01$, the second best option is to set $s^* = 0.01$. In contrast, for $\sigma = 8$ and $r = 0.2$, the policy maker should set $s^* = 0.15$ which is lower than $r = 0.20$.

Using equation (10), optimal tax rate can be obtained as

$$\tau^* = \frac{\alpha}{L\sigma}. \quad (16)$$

Given $r \in (0,1)$, it is easy to verify that $s^* > 0$ from equation (15) which also implies a positive optimal tax rate as well.^[11]

To find the effect of optimal government policy on welfare, it is enough to solve the following equation:

$$V(s = s^*, \tau = \tau^*) - V(s = 0, \tau = 0) = \alpha \left[\log\left(\frac{\sigma}{\sigma-1}\right) - \frac{1}{\sigma} \right] > 0. \quad (17)$$

Given $\alpha > 0$, V is always positive.

Thus, government reform on implementing optimal credit subsidy always improves welfare.

2. Some subsidy levels can decrease welfare compared to a no policy case.

In order to find the welfare decreasing credit subsidy level, we investigate the following equation:

$$W = V(s > 0, \tau > 0) - V(s = 0, \tau = 0) = \alpha \left[\frac{1-\sigma}{\sigma} \frac{s}{1+r-s} - \log\left(\frac{1+r-s}{1+r}\right) \right]. \quad (18)$$

Taking the derivative of W with respect to s we have

$$\frac{\partial W}{\partial s} = \frac{\alpha}{1+r-s} \left[\frac{1-\sigma}{\sigma} \frac{1+r}{1+r-s} + 1 \right]. \quad (19)$$

By setting $\frac{\partial W}{\partial s} = 0$, we get $s^* = \frac{r+1}{\sigma} > 0$ which is, of course, the same with equation (15). If $0 \leq s < s^*$, then $\frac{\partial W}{\partial s} > 0$ and if $s > s^*$ then $\frac{\partial W}{\partial s} < 0$. Moreover, there exists a unique $\hat{s} > s^*$ which makes $W = 0$ in equation (18). Depending on parameters $\sigma > 1$ and $r \in (0,1)$, W takes both negative and positive values. If $s < \hat{s}$ then $W > 0$ and if $s > \hat{s}$ then $W < 0$.

^[11] In order to guarantee that $s^* < r$, we assume that $\frac{r+1}{\sigma} < r$. Moreover, under assumption $\sigma > 1$ and $r \in (0,1)$, τ^* is always positive and $\tau^* < 1 - \frac{\alpha}{L}$ if $\frac{\alpha}{L-\alpha} < \sigma$. Since $\sigma > 1$, $2\alpha < L$ is sufficient to have an optimal tax rate which is smaller than the upper limit of tax.

Figure 1:
Graph of W as a Function of s

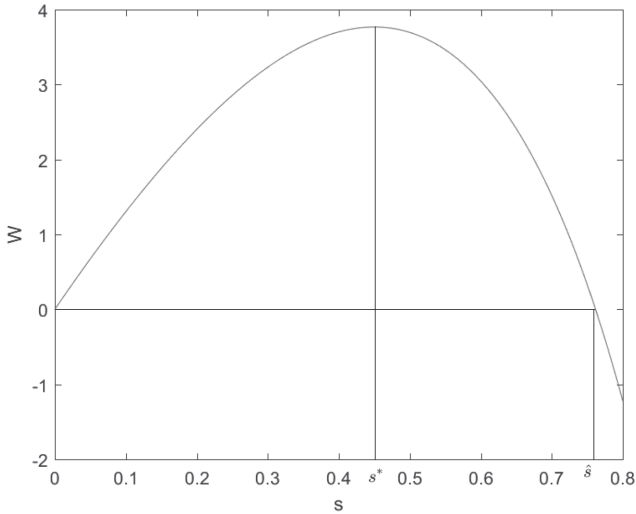


Figure 1 shows a numerical example to visualize the above arguments,^[12] As a result, a positive but not an optimal credit subsidy, compared to no policy, can either increase or decrease the welfare.

3. Market interest rates are crucial in determining the optimal credit subsidy.

Equation (15) shows the crucial relation between the optimal credit subsidy and the market interest rate. Since optimal credit subsidy depends on elasticity of substitution across goods and market interest rate, we do comparative statics with respect to these parameters. From equation (15), it is obvious that a rise in σ has a negative effect on s^* , while an increase in r increases s^* .

$$\frac{\partial s^*}{\partial r} = \frac{1}{\sigma} > 0. \quad (20)$$

Conclusion

This paper improves upon the existing literature by showing the importance of optimal credit subsidy in improving welfare. On the other hand, implementing the positive but not the optimal credit subsidy level can either increase or decrease welfare. However, the main contribution of this study is to show that the market interest rate is a crucial indicator in determining an optimal credit subsidy level. Hence, policy makers should be very careful about the subsidy level since some levels of subsidies can decrease welfare.

^[12]Figure 1 is drawn for $\alpha = 100$, $\sigma = 4$ and $r = 0.8$.

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